1. A biased coin is weighted such that the probability of obtaining a head is $\frac{4}{7}$. The coin is tossed 6 times and X denotes the number of heads observed. Find the value of the ratio $\frac{\mathrm{P}(X=3)}{\mathrm{P}(X=2)}$.

(Total 4 marks)

2. Casualties arrive at an accident unit with a mean rate of one every 10 minutes.
Assume that the number of arrivals can be modelled by a Poisson distribution.

(a) Find the probability that there are no arrivals in a given half hour period.

(3)

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period.

(3)

(c) Six nurses work consecutive two hour periods between 8am and 8pm.
Find the probability that no more than three nurses have to attend to less than ten casualties during their working period.

(4)

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties.

(5)
(Total 15 marks)

3. Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(1)

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(3)

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water is greater than 0.99.

(4)

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(6)

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $P(X = 2) = P(X = 3) + P(X = 4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

(6)
(Total 20 marks)

4. Over a one month period, Ava and Sven play a total of n games of tennis.

The probability that Ava wins any game is 0.4. The result of each game played is independent of any other game played.

Let X denote the number of games won by Ava over a one month period.

(a) Find an expression for $P(X = 2)$ in terms of n.

(3)

(b) If the probability that Ava wins two games is 0.121 correct to three decimal places, find the value of n.

(3)
(Total 6 marks)

5. A factory makes wine glasses. The manager claims that on average 2% of the glasses are imperfect. A random sample of 200 glasses is taken and 8 of these are found to be imperfect.

Test the manager's claim at a 1% level of significance using a one-tailed test.

(Total 7 marks)

6. (a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. What is the probability that a box is underweight?

(2)

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.

(6)

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight.

(3)
(Total 11 marks)