1. (a) (i) $P(A \cup B) = P(A) + P(B) = 0.7$ A1

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ (M1)
$= P(A) + P(B) - P(A)P(B)$ (M1)
$= 0.3 + 0.4 - 0.12 = 0.58$ A1

(b) $P(A \cap B) = P(A) + P(B) - P(A \cup B)$
$= 0.3 + 0.4 - 0.6 = 0.1$ A1

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ (M1)

$= \dfrac{0.1}{0.4} = 0.25$ A1

[7]

2. (a) $P(X < 30) = 0.4$
$P(X < 55) = 0.9$
or relevant sketch (M1)

given $Z = \dfrac{X - \mu}{\sigma}$

$P(Z < z) = 0.4 \Rightarrow \dfrac{30 - \mu}{\sigma} = -0.253...$ (A1)

$P(Z < z) = 0.9 \Rightarrow \dfrac{55 - \mu}{\sigma} = 1.28...$ (A1)

$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma$ M1
$\sigma = 16.3, \mu = 34.1$ A1

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two *M* marks.

(b) $X \sim N(34.12..., 16.28...^2)$
late to school $\Rightarrow X > 60$
$P(X > 60) = 0.056...$ (A1)
number of students late $= 0.0560... \times 1200$ (M1)
$= 67$ (to nearest integer) A1

Note: Accept 62 for use of 34 and 16.

(c) $P(X > 60 \mid X > 30) = \dfrac{P(X > 60)}{P(X > 30)}$ M1

$= 0.0935$ (accept anything between 0.093 and 0.094)A1

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

(d) let L be the random variable of the number of students who leave school in a 30 minute interval

since $24 \times 30 = 720$ A1

$L \sim \text{Po}(720)$

$\text{P}(L \geq 700) = 1 - \text{P}(L \leq 699)$ (M1)

$= 0.777$ A1

Note: Award M1A0 for $\text{P}(L > 700) = 1 - \text{P}(L \leq 700)$ (this leads to 0.765).

(e) (i) $Y \sim \text{B}(200, 0.7767\ldots)$ (M1)

$\text{E}(Y) = 200 \times 0.7767\ldots = 155$ A1

Note: On FT, use of 0.765 will lead to 153.

(ii) $\text{P}(Y > 150) = 1 - \text{P}(Y \leq 150)$ (M1)

$= 0.797$ A1

Note: Accept 0.799 from using rounded answer.

Note: On FT, use of 0.765 will lead to 0.666.

[17]

3. (a) $\int_0^1 a\text{e}^{-ax}\,\text{d}x = 1 - \frac{1}{\sqrt{2}}$ M1A1

$\left[-\text{e}^{-ax}\right]_0^1 = 1 - \frac{1}{\sqrt{2}}$ M1A1

$-\text{e}^{-a} + 1 = 1 - \frac{1}{\sqrt{2}}$ A1

Note: Accept e^0 instead of 1.

$\text{e}^{-a} = \frac{1}{\sqrt{2}}$

$\text{e}^{a} = \sqrt{2}$

$a = \ln 2^{\frac{1}{2}}$ $\left(\text{accept } -a = \ln 2^{-\frac{1}{2}}\right)$ A1

$a = \frac{1}{2}\ln 2$ AG

(b) $\int_0^M a\mathrm{e}^{-ax}\,\mathrm{d}x = \frac{1}{2}$ M1A1

$\left[-\mathrm{e}^{-ax}\right]_0^M = \frac{1}{2}$

$-\mathrm{e}^{-Ma} + 1 = \frac{1}{2}$

$\mathrm{e}^{-Ma} = \frac{1}{2}$ A1

$Ma = \ln 2$

$M = \frac{\ln 2}{a} = 2$ A1

(c) $\mathrm{P}(1 < X < 3) = \int_1^3 a\mathrm{e}^{-ax}\,\mathrm{d}x$ M1A1

$= -\mathrm{e}^{-3a} + \mathrm{e}^{-a}$ A1

$\mathrm{P}(X < 3|X > 1) = \frac{\mathrm{P}(1 < X < 3)}{\mathrm{P}(X > 1)}$ M1A1

$= \frac{-\mathrm{e}^{-3a} + \mathrm{e}^{-a}}{1 - \mathrm{P}(X < 1)}$ A1

$= \frac{-\mathrm{e}^{-3a} + \mathrm{e}^{-a}}{\frac{1}{\sqrt{2}}}$ A1

$= \sqrt{2}\,(-\mathrm{e}^{-3a} + \mathrm{e}^{-a})$

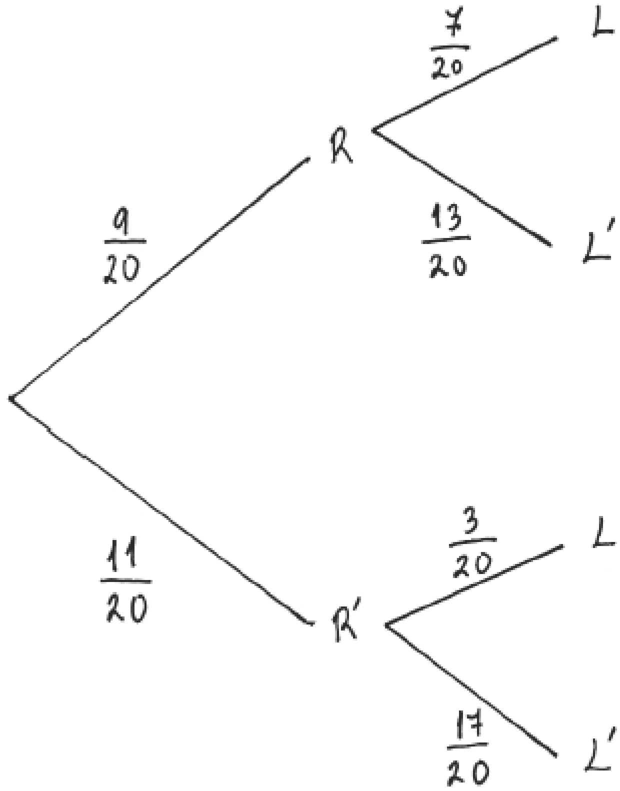
$= \sqrt{2}\left(-2^{-\frac{3}{2}} + 2^{-\frac{1}{2}}\right)$ A1

$= \frac{1}{2}$ A1

Note: Award full marks for $\mathrm{P}(X < 3 \mid X > 1) = \mathrm{P}(X < 2) = \frac{1}{2}$ or quoting properties of exponential distribution.

[20]

4.



(A1)

$\mathrm{P}(R' \cap L) = \frac{11}{20} \times \frac{3}{20}$ A1

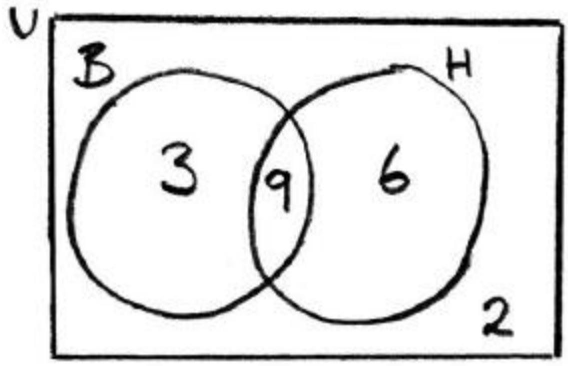
$\mathrm{P}(L) = \frac{9}{20} \times \frac{7}{20} + \frac{11}{20} \times \frac{3}{20}$ A1

$P(R'|L) = \frac{\mathrm{P}(R' \cap L)}{\mathrm{P}(L)}$ (M1)

$= \frac{33}{96} \left(= \frac{11}{32} \right)$ A1

[5]

5. (a)



A1A1

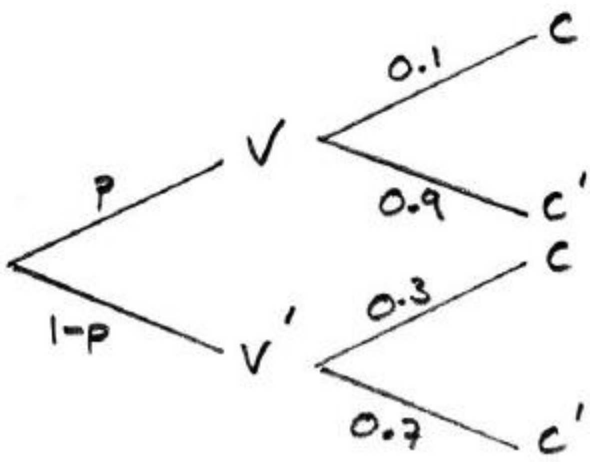
Note: Award A1 for a diagram with two intersecting regions and at least the value of the intersection.

(b) $\frac{9}{20}$ A1

(c) $\frac{9}{12}\left(=\frac{3}{4}\right)$ A1

[4]

6. (a)



using the law of total probabilities: (M1)

$0.1p + 0.3(1-p) = 0.22$ A1

$0.1p + 0.3 - 0.3p = 0.22$

$0.2p = 0.08$

$p = \frac{0.08}{0.2} = 0.4$

$p = 40\%$ (accept 0.4) A1

(b) required probability $= \frac{0.4 \times 0.1}{0.22}$ M1

$= \frac{2}{11}$ (0.182) A1

[5]

7. $P(M \mid G) = \frac{P(M \cap G)}{P(G)}$ (M1)

$= \frac{0.2 \times 0.9}{0.2 \times 0.9 + 0.8 \times 0.7}$ M1A1A1

$= \frac{0.18}{0.74}$

$= \frac{9}{37}$ A1

[5]

8. the waiting time, $X \sim N(18, 4^2)$

(a) $P(X > 25) = 0.0401$ (M1)A1

(b) $P(X < 20 \mid X > 15)$

$= \frac{P(15 < X < 20)}{P(X > 15)}$ (A1)

Note: Only one of the above A1 marks can be implied.

$= \frac{0.4648...}{0.7733...} = 0.601$ (M1)A1

[6]

9. **EITHER**

Using $P(A \mid B) = \frac{P(A \cap B)}{P(B)}$ (M1)

$0.6P(B) = P(A \cap B)$ A1

Using $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ to obtain
$0.8 = 0.6 + P(B) - P(A \cap B)$ A1

Substituting $0.6P(B) = P(A \cap B)$ into above equation M1

OR

As P(A | B) = P(A) then A and B are independent events M1R1

Using P($A \cup B$) = P(A) + P(B) – P(A) × P(B) A1

to obtain 0.8 = 0.6 + P(B) – 0.6 × P(B) A1

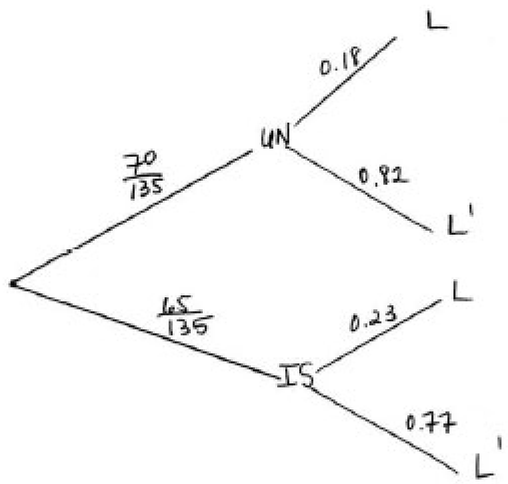
THEN

0.8 = 0.6 + 0.4P(B) A1

P(B) = 0.5 A1 N1

[6]

10. METHOD 1



(M1)

Let P(I) be the probability of flying IS Air, P(U) be the probability flying UN Air and P(L) be the probability of luggage lost.

$$\mathrm{P}(I \mid L) = \frac{\mathrm{P}(I \cap L)}{\mathrm{P}(L)}$$

$$\left(\text{or Bayes' formula}, \mathrm{P}(I \mid L) = \frac{\mathrm{P}(L \mid I)\mathrm{P}(I)}{\mathrm{P}(L \mid I)\mathrm{P}(I) + \mathrm{P}(L \mid U)\mathrm{P}(U)}\right)$$ (M1)

$$= \frac{0.23 \times \frac{65}{135}}{0.18 \times \frac{70}{135} + 0.23 \times \frac{65}{135}}$$ A1A1A1

$$= \frac{299}{551} \quad (= 0.543, \text{accept } 0.542)$$ A1

METHOD 2

Expected number of suitcases lost by UN Air is $0.18 \times 70 = 12.6$ M1A1

Expected number of suitcases lost by IS Air is $0.23 \times 65 = 14.95$ A1

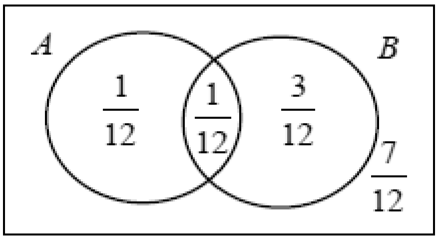
$P(I \mid L) = \frac{14.95}{12.6 + 14.95}$ M1A1

$= 0.543$ A1

[6]

11. $P(A \cap B) = P(A) + P(B) - P(A \cup B)$ M1

$= \frac{2}{12} + \frac{4}{12} - \frac{5}{12} = \frac{1}{12}$ A1



M1A1

$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{\frac{7}{12}}{\frac{8}{12}} = \frac{7}{8}$ M1A1

[6]

12. (a) $P(RR) = \left(\frac{2}{5}\right)\left(\frac{1}{4}\right)$ (M1)

$= \frac{1}{10}$ A1 N2

(b) $P(RR) = \frac{4}{4+n} \times \frac{3}{3+n} = \frac{2}{15}$ A1

Forming equation $12 \times 15 = 2(4 + n)(3 + n)$ (M1)

$12 + 7n + n^2 = 90$ A1

$\Rightarrow n^2 + 7n - 78 = 0$ A1

$n = 6$ AG N0

(c) **EITHER**

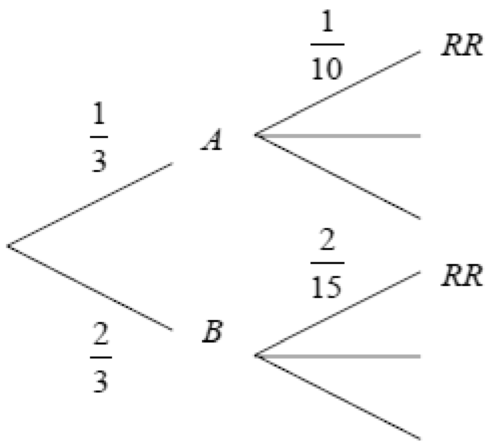
P(A) = $\frac{1}{3}$ P(B) $\frac{2}{3}$ A1

P(RR) = P($A \cap RR$) + P($B \cap RR$) (M1)

$= \left(\frac{1}{3}\right)\left(\frac{1}{10}\right) + \left(\frac{2}{3}\right)\left(\frac{2}{15}\right)$

$= \frac{11}{90}$ A1 N2

OR



A1

P(RR) = $\frac{1}{3} \times \frac{1}{10} + \frac{2}{3} \times \frac{2}{15}$ M1

$= \frac{11}{90}$ A1 N2

(d) P(1or 6) = P(A) M1

$P(A \mid RR) = \frac{P(A \cap RR}{P(RR)}$ (M1)

$= \frac{\left[\left(\frac{1}{3}\right)\left(\frac{1}{10}\right)\right]}{\frac{11}{90}}$ M1

$= \frac{3}{11}$ A1 N2

[13]

13. (a) P($3 \le X \le 5$) = P($X \le 5$) – P($X \le 2$) (M1)
= 0.547 A1 N2

(b) P($X \ge 3$) = 1 – P($X \le 2$) (M1)
= 0.762 A1 N2

(c) $P(3 \le X \le 5 \mid X \ge 3) = \dfrac{P(3 \le X \le 5)}{P(X \ge 3)} \left(= \dfrac{0.547}{0.762}\right)$ (M1)

$= 0.718$ A1 N2

[6]