1. **METHOD 1**

$1 + \mathrm{i}$ is a zero $\Rightarrow$ $1 - \mathrm{i}$ is a zero	(A1)
$1 - 2\mathrm{i}$ is a zero $\Rightarrow$ $1 + 2\mathrm{i}$ is a zero	(A1)
$(x - (1 - \mathrm{i}))(x - (1 + \mathrm{i})) = (x^2 - 2x + 2)$	(M1)A1
$(x - (1 - 2\mathrm{i}))(x - (1 + 2\mathrm{i})) = (x^2 - 2x + 5)$	A1
$p(x) = (x^2 - 2x + 2)(x^2 - 2x + 5)$	M1
$\quad = x^4 - 4x^3 + 11x^2 - 14x + 10$	A1
$a = -4$, $b = 11$, $c = -14$, $d = 10$	

METHOD 2

$p(1 + \mathrm{i}) = -4 + (-2 + 2\mathrm{i})a + (2\mathrm{i})b + (1 + \mathrm{i})c + d$	M1
$p(1 + \mathrm{i}) = 0 \Rightarrow \begin{cases} -4 - 2a + c + d = 0 \\ 2a + 2b + c = 0 \end{cases}$	M1A1A1
$p(1 - 2\mathrm{i}) = -7 + 24\mathrm{i} + (-11 + 2\mathrm{i})a + (-3 - 4\mathrm{i})b + (1 - 2\mathrm{i})c + d$	
$p(1 - 2\mathrm{i}) = 0 \Rightarrow \begin{cases} -7 - 11a - 3b + c + d = 0 \\ 24 + 2a - 4b - 2c = 0 \end{cases}$	A1
$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 & 1 \\ 2 & 2 & 1 & 0 \\ -11 & -3 & 1 & 1 \\ 2 & -4 & -2 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 4 \\ 0 \\ 7 \\ -24 \end{pmatrix} = \begin{pmatrix} -4 \\ 11 \\ -14 \\ 10 \end{pmatrix}$	M1A1
$a = -4$, $b = 11$, $c = -14$, $d = 10$	

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2. (a) $1 - \mathrm{i}\sqrt{3}$ A1

(b) **EITHER**

$(z - (1 + \mathrm{i}\sqrt{3}))(z - (1 - \mathrm{i}\sqrt{3})) = z^2 - 2z + 4$	(M1)A1
$p(z) = (z - 2)(z^2 - 2z + 4)$	(M1)
$= z^3 - 4z^2 + 8z - 8$	A1
therefore $b = -4$, $c = 8$, $d = -8$	

OR

relating coefficients of cubic equations to roots	
$-b = 2 + 1 + \mathrm{i}\sqrt{3} + 1 - \mathrm{i}\sqrt{3} = 4$	M1
$c = 2(1 + \mathrm{i}\sqrt{3}) + 2(1 - \mathrm{i}\sqrt{3}) + (1 + \mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = 8$	
$-d = 2(1 + \mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = 8$	
$b = -4$, $c = 8$, $d = -8$	A1A1A1

(c) $z_2 = 2e^{\frac{i\pi}{3}}, z_3 = 2e^{-\frac{i\pi}{3}}$ A1A1A1

Note: Award A1 for modulus,
A1 for each argument.

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3. (a) $i^4 - 5i^3 + 7i^2 - 5i + 6 = 1 + 5i - 7 - 5i + 6$ M1A1
$= 0$ AG N0

(b) i root $\Rightarrow$ –i is second root (M1)A1
moreover, $x^4 - 5x^3 + 7x^2 - 5x + 6 = (x - i)(x + i)\,q(x)$
where $q(x) = x^2 - 5x + 6$
finding roots of $q(x)$
the other two roots are 2 and 3 A1A1

Note: Final A1A1 is independent of previous work.

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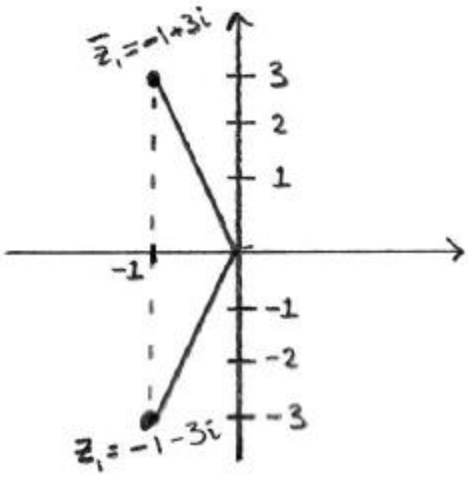
4. (a) one root is $-1 - 3i$ A1



distance between roots is 6, implies height is 3 (M1)A1

EITHER

$-1 + 3 = 2 \Rightarrow$ third root is 2 A1

OR

$-1 - 3 = -4 \Rightarrow$ third root is -4 A1

(b) **EITHER**

$(z - (-1 + 3i))(z - (-1 - 3i))\ (z - 2) = 0$ M1

$\Rightarrow\ (z^2 + 2z + 10)(z - 2) = 0$ (A1)

$z^3 + 6z - 20 = 0$ A1

$a = 0$, $b = 6$ and $c = -20$

OR

$(z - (-1 + 3i))(z - (-1 - 3i))\ (z + 4) = 0$ M1

$\Rightarrow\ (z^2 + 2z + 10)(z + 4) = 0$ (A1)

$z^3 + 6z^2 + 18z + 40 = 0$ A1

$a = 6$, $b = 18$ and $c = 40$

[7]

5. (a) $(x - 1)(x^4 + x^3 + x^2 + x + 1)$

$= x^5 + x^4 + x^3 + x^2 + x - x^4 - x^3 - x^2 - x - 1$ (M1)

$= x^5 - 1$ A1

(b) b is a root

$f(b) = 0$

$b^5 = 1$ M1

$b^5 - 1 = 0$ A1

$(b - 1)(b^4 + b^3 + b^2 + b + 1) = 0$

$b \neq 1$ R1

$1 + b + b^2 + b^3 + b^4 = 0$ as shown. AG

(c) (i) $u + v = b^4 + b^3 + b^2 + b = -1$ A1

$uv = (b + b^4)(b^2 + b^3) = b^3 + b^4 + b^6 + b^7$ A1

Now $b^5 = 1$ (A1)

Hence $uv = b^3 + b^4 + b + b^2 = -1$ A1

Hence $u + v = uv = -1$ AG

(ii) $(u-v)^2 = (u^2+v^2) - 2uv$ (M1)

$= ((u+v)^2 - 2uv) - 2uv \quad (= (u+v)^2 - 4uv)$ (M1)A1

Given $u - v > 0$

$u - v = \sqrt{(u+v)^2 - 4uv}$

$= \sqrt{(-1)^2 - 4(-1)}$

$= \sqrt{1+4}$ A1

$= \sqrt{5}$ AG

Note: Award A0 unless an indicator is given that $u - v = -\sqrt{5}$ is invalid.

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6. $2 + \text{i}$ is a root $\Rightarrow$ $2 - \text{i}$ is a root R1

$[x - (2+\text{i})]\,[x - (2-\text{i})]$ are factors M1

$= x^2 - (2-\text{i})\,x - (2+\text{i})\,x + (2+\text{i})(2-\text{i})$

$= x^2 - 2x + \text{i}x - 2x - \text{i}x + (4+1)$ (A1)

$= x^2 - 4x + 5$ A1

Hence $x - 2$ is a factor $\Rightarrow$ 2 is a root R1

[5]

7. (a) Let $p = 2$, $\Rightarrow$ $8 + 4 - 10 - 2 = 0$ M1

Since this fits $p = 2$ is a solution. R1

(b) $p^3 + p^2 - 5p - 2 = (p-2)(p^2 + ap + b)$

$= p^3 + ap^2 + bp - 2p^2 - 2ap - 2b$ M1A1

$= p^3 + p^2(a-2) + p(b-2a) - 2b$

Equate constants $\Rightarrow$ $-2 = -2b$

$b = 1$ A1

Equate coefficients of p^2 $\Rightarrow$ $a - 2 = 1$

$a = 3$ A1

(c) $p^2 + 3p + 1 = 0$ M1

$p = \dfrac{-3 \pm \sqrt{9-4}}{2} = \dfrac{-3 \pm \sqrt{5}}{2}$ A1A1

(d) (i) Arithmetic sequence: $1,\ 1+p,\ 1+2p,\ 1+3p$ A1

Geometric sequence: $1,\ p,\ p^2,\ p^3$ A1

(ii) $(1 + 2p) + (1 + 3p) = p^2 + p^3$ M1A1

$\Rightarrow p^3 + p^2 - 5p - 2 = 0$ A1

Therefore, from part (i), $p = 2$, $p = \dfrac{-3 \pm \sqrt{5}}{2}$ R1

(iii) The sum to infinity of a geometric series exists if $|p| < 1$. R1

Hence, $p = \dfrac{-3 + \sqrt{5}}{2}$ is the only such number. A1

(iv) The sum of the first 20 terms of the arithmetic series can be found by applying the sum formula

$S_{20} = 10(2a + 19d) = 10(2 + 19p)$ M1A1

So, $S_{20} = 10\left(2 + 19\left(\dfrac{\sqrt{5} - 3}{2}\right)\right) = -265 + 95\sqrt{5}$ A1A1A1

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