1. Consider the function $f(x) = \dfrac{\ln x}{x}$, $0 < x < e^2$.

(a) (i) Solve the equation $f'(x) = 0$.

(ii) Hence show the graph of f has a local maximum.

(iii) Write down the range of the function f.

(5)

(b) Show that there is a point of inflexion on the graph and determine its coordinates.

(5)

(c) Sketch the graph of $y = f(x)$, indicating clearly the asymptote, x-intercept and the local maximum.

(3)

(d) Now consider the functions $g(x) = \dfrac{\ln|x|}{x}$ and $h(x) = \dfrac{\ln|x|}{|x|}$, where $0 < |x| < e^2$.

(i) Sketch the graph of $y = g(x)$.

(ii) Write down the range of g.

(iii) Find the values of x such that $h(x) > g(x)$.

(6)
(Total 19 marks)

2. The quadratic function $f(x) = p + qx - x^2$ has a maximum value of 5 when $x = 3$.

(a) Find the value of p and the value of q.

(4)

(b) The graph of $f(x)$ is translated 3 units in the positive direction parallel to the x-axis. Determine the equation of the new graph.

(2)
(Total 6 marks)

3. Consider the function $f(x) = x^3 - 3x^2 - 9x + 10$, $x \in \mathbb{R}$.

(a) Find the equation of the straight line passing through the maximum and minimum points of the graph $y = f(x)$.

(4)

(b) Show that the point of inflexion of the graph $y = f(x)$ lies on this straight line.

(2)
(Total 6 marks)

4. Consider $f(x) = \dfrac{x^2 - 5x + 4}{x^2 + 5x + 4}$.

(a) Find the equations of all asymptotes of the graph of f.

(4)

(b) Find the coordinates of the points where the graph of f meets the x and y axes.

(2)

(c) Find the coordinates of

(i) the maximum point and justify your answer;

(ii) the minimum point and justify your answer.

(10)

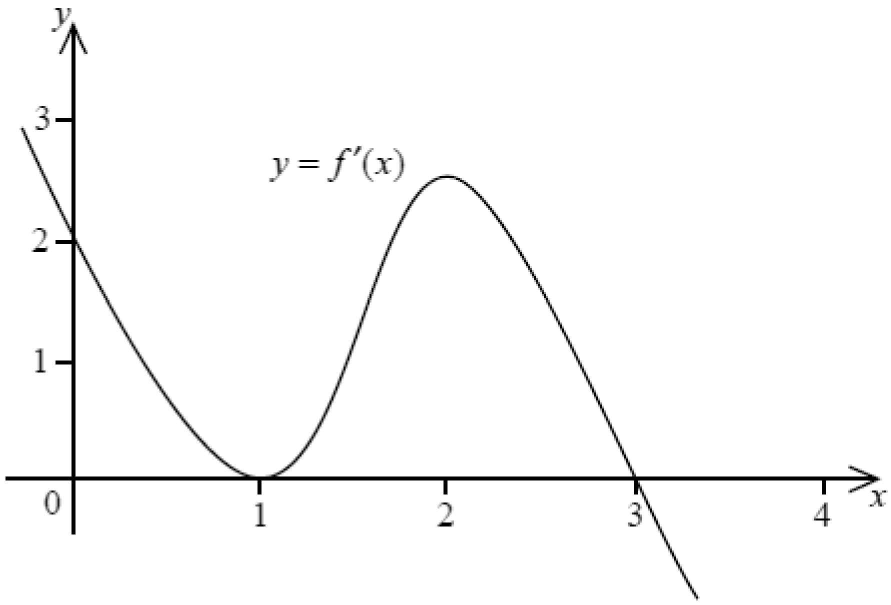
(d) Sketch the graph of f, clearly showing all the features found above.

(3)

(e) **Hence**, write down the number of points of inflexion of the graph of f.

(1)
(Total 20 marks)

5. The diagram below shows a sketch of the gradient function $f'(x)$ of the curve $f(x)$.



On the graph below, sketch the curve $y = f(x)$ given that $f(0) = 0$. Clearly indicate on the graph any maximum, minimum or inflexion points.

(Total 5 marks)

6. The function f is defined by $f(x) = x\,e^{2x}$.

It can be shown that $f^{(n)}(x) = (2^n x + n\,2^{n-1})\,e^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$.

(a) By considering $f^{(n)}(x)$ for $n = 1$ and $n = 2$, show that there is one minimum point P on the graph of f, and find the coordinates of P.

(7)

(b) Show that f has a point of inflexion Q at $x = -1$.

(5)

(c) Determine the intervals on the domain of f where f is

(i) concave up;

(ii) concave down.

(2)

(d) Sketch f, clearly showing any intercepts, asymptotes and the points P and Q.

(4)

(e) Use mathematical induction to prove that $f^{(n)}(x) = (2^n x + n2^{n-1})\,e^{2x}$ for all $n \in \mathbb{Z}^+$, where $f^{(n)}(x)$ represents the n^{th} derivative of $f(x)$.

(9)
(Total 27 marks)

7. A family of cubic functions is defined as $f_k(x) = k^2x^3 - kx^2 + x$, $k \in \mathbb{Z}^+$.

(a) Express in terms of k

(i) $f'_k(x)$ and $f''_k(x)$;

(ii) the coordinates of the points of inflexion P_k on the graphs of f_k.

(6)

(b) Show that all P_k lie on a straight line and state its equation.

(2)

(c) Show that for all values of k, the tangents to the graphs of f_k at P_k are parallel, and find the equation of the tangent lines.

(5)
(Total 13 marks)

8. Consider the curve with equation $f(x) = e^{-2x^2}$ for $x < 0$.

Find the coordinates of the point of inflexion and justify that it is a point of inflexion.

(Total 7 marks)

9. The function f is defined by $f(x) = (\ln(x - 2))^2$. Find the coordinates of the point of inflexion of f.

(Total 9 marks)

10. It is given that

$$f(x) = \frac{18(x-1)}{x^2},\ f'(x) = \frac{18(2-x)}{x^3}, \text{ and } f''(x) = \frac{36(x-3)}{x^4},\ x \in \mathbb{R},\ x \neq 0.$$

(a) Find

(i) the zero(s) of $f(x)$;

(ii) the equations of the asymptotes;

(iii) the coordinates of the local maximum and justify it is a maximum;

(iv) the interval(s) where $f(x)$ is concave up.

(9)

(b) Hence sketch the graph of $y = f(x)$.

(5)
(Total 14 marks)

11. The function f is defined on the domain $x \geq 1$ by $f(x) = \dfrac{\ln x}{x}$.

(a) (i) Show, by considering the first and second derivatives of f, that there is one maximum point on the graph of f.

(ii) State the **exact** coordinates of this point.

(iii) The graph of f has a point of inflexion at P. Find the x-coordinate of P.

(12)

Let R be the region enclosed by the graph of f, the x-axis and the line $x = 5$.

(b) Find the **exact** value of the area of R.

(6)
(Total 18 marks)

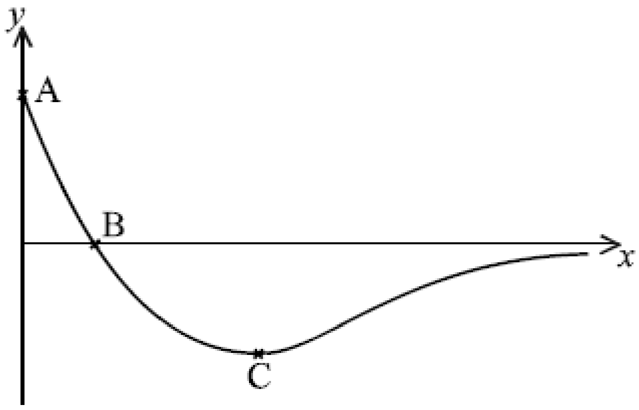
12. (a) Find the root of the equation $e^{2-2x} = 2e^{-x}$ giving the answer as a logarithm.

(4)

(b) The curve $y = e^{2-2x} - 2e^{-x}$ has a minimum point. Find the coordinates of this minimum.

(7)

(c) The curve $y = e^{2-2x} - 2e^{-x}$ is shown below.



Write down the coordinates of the points A, B and C.

(3)

(d) Hence state the set of values of k for which the equation $e^{2-2x} - 2e^{-x} = k$ has two distinct positive roots.

(2)
(Total 16 marks)

13. The function f is defined on the domain $x \geq 0$ by $f(x) = \dfrac{x^2}{e^x}$.

(a) Find the maximum value of $f(x)$, and justify that it is a maximum.

(10)

(b) Find the x coordinates of the points of inflexion on the graph of f.

(3)

(c) Evaluate $\int_0^1 f(x)\,dx$.

(8)
(Total 21 marks)