CHAPTER 3 REVIEW – FOR SLs ONLY

1. Find the coordinates of the point on the graph of $y = x^2 - x$ at which the tangent is parallel to the line $y = 5x$.

(Total 4 marks)

2. Find the equation of the normal to the curve with equation

$$y = x^3 + 1$$

at the point (1, 2).

(Total 4 marks)

3. Let $f'(x) = -24x^3 + 9x^2 + 3x + 1$.

(a) There are two points of inflexion on the graph of f. Write down the x-coordinates of these points.

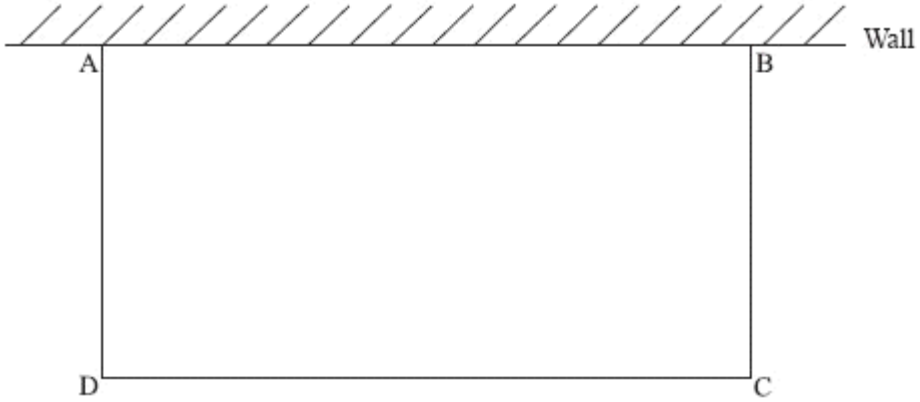
(3)

(b) Let $g(x) = f''(x)$. Explain why the graph of g has no points of inflexion.

(2)

(Total 5 marks)

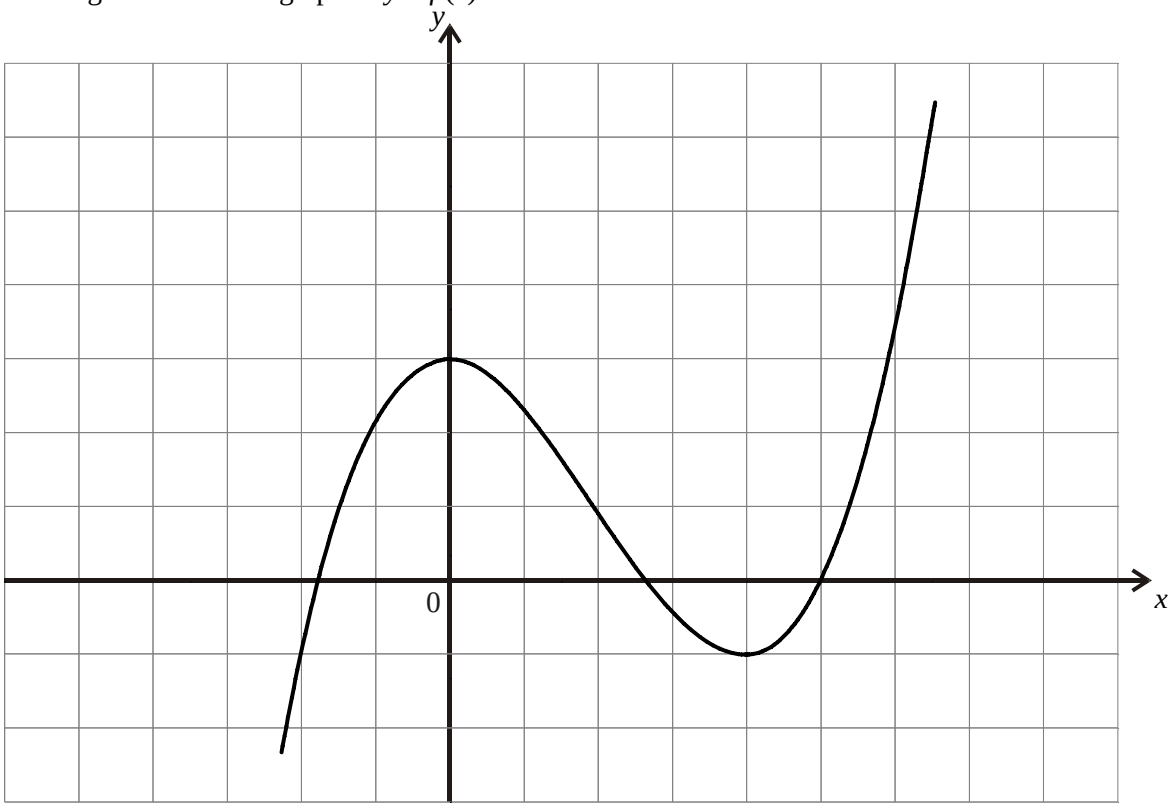
4. The following diagram shows a rectangular area ABCD enclosed on three sides by 60 m of fencing, and on the fourth by a wall AB.



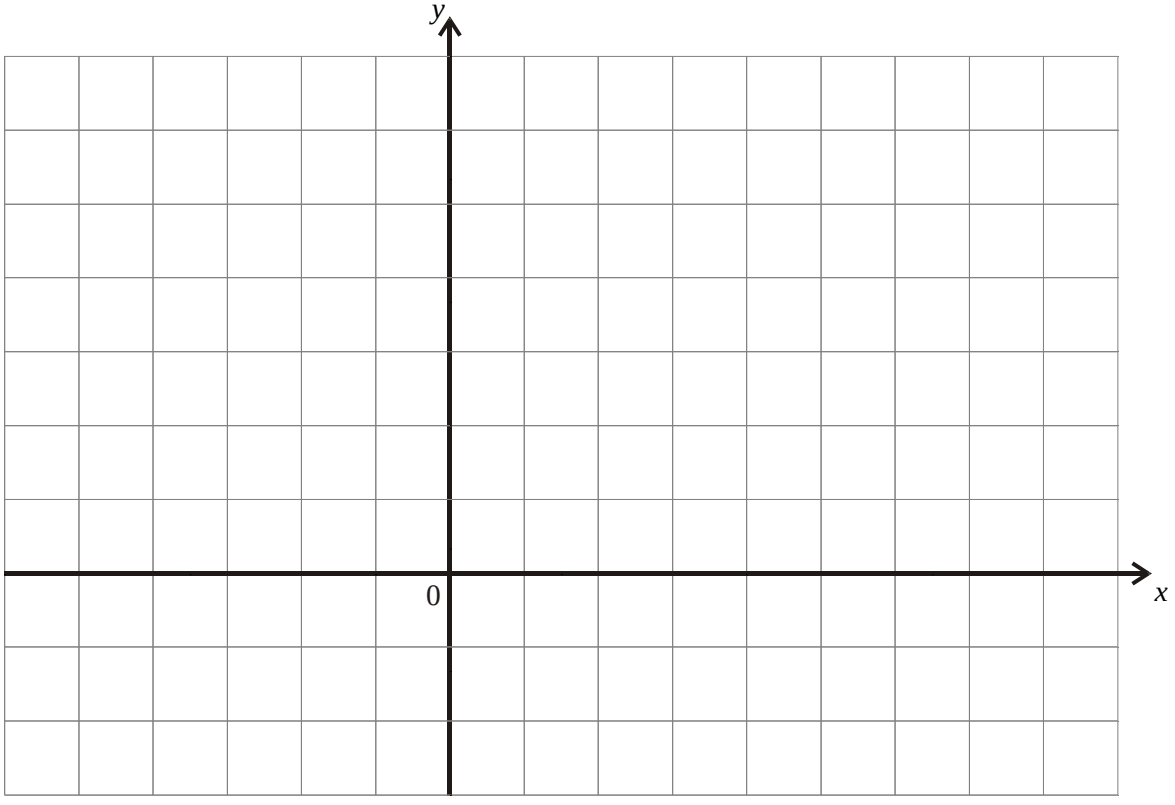
Find the width of the rectangle that gives its maximum area.

(Total 6 marks)

5. The diagram shows the graph of $y = f(x)$.



On the grid below sketch the graph of $y = f'(x)$.



(Total 6 marks)

6. The graph of $y = x^3 - 10x^2 + 12x + 23$ has a maximum point between $x = -1$ and $x = 3$. Find the coordinates of this maximum point.

(Total 6 marks)

7. Let $f(x) = x^3 - 3x^2 - 24x + 1$.
The tangents to the curve of f at the points P and Q are parallel to the x-axis, where P is to the left of Q.

(a) Calculate the coordinates of P and of Q.

Let N_1 and N_2 be the normals to the curve at P and Q respectively.

(b) Write down the coordinates of the points where

(i) the tangent at P intersects N_2;

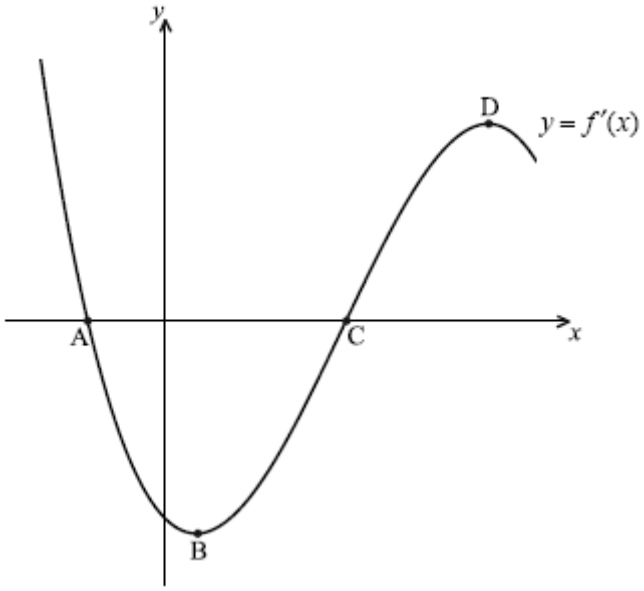
(ii) the tangent at Q intersects N_1.

(Total 6 marks)

8. Consider the function $f(x) = 4x^3 + 2x$. Find the equation of the normal to the curve of f at the point where $x = 1$.

(Total 6 marks)

9. The diagram shows part of the graph of $y = f'(x)$. The x-intercepts are at points A and C.
There is a minimum at B, and a maximum at D.



(a) (i) Write down the value of $f'(x)$ at C.

(ii) **Hence**, show that C corresponds to a minimum on the graph of f, *i.e.* it has the same x-coordinate.

(3)

(b) Which of the points A, B, D corresponds to a maximum on the graph of f?

(1)

(c) Show that B corresponds to a point of inflexion on the graph of f.

(3)

(Total 7 marks)

10. A farmer wishes to create a rectangular enclosure, ABCD, of area 525 m^2, as shown below.

The fencing used for side AB costs \$11 per metre. The fencing for the other three sides costs \$3 per metre. The farmer creates an enclosure so that the cost is a minimum.
Find this minimum cost.

(Total 7 marks)

11. The function f is defined by $f : x \mapsto -0.5x^2 + 2x + 2.5$.
Let N be the normal to the curve at the point where the graph intercepts the y-axis.

(a) Show that the equation of N may be written as $y = -0.5x + 2.5$.

(4)

(b) Find the coordinates of the other point of intersection of the normal and the curve.

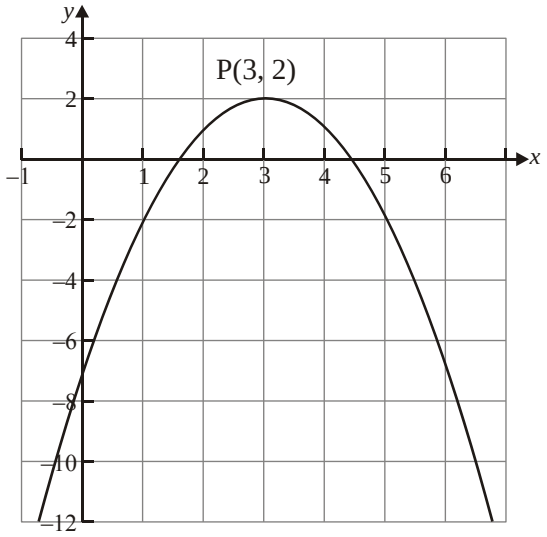
(5)

(c) Let R be the region enclosed between the curve and N. Find the area of R.

(4)

(Total 13 marks)

12. The function $f(x)$ is defined as $f(x) = -(x - h)^2 + k$. The diagram below shows part of the graph of $f(x)$.
The maximum point on the curve is P (3, 2).



(a) Write down the value of
(i) h;
(ii) k.

(2)

(b) Show that $f(x)$ can be written as $f(x) = -x^2 + 6x - 7$.

(1)

(c) Find $f'(x)$.

(2)

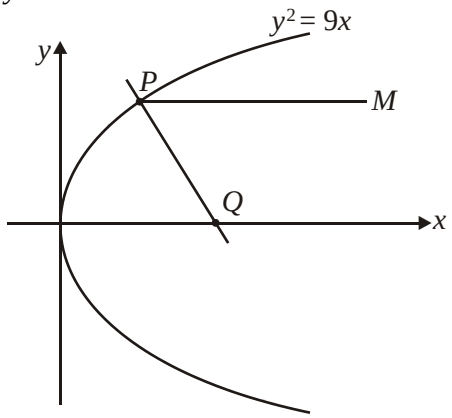
The point Q lies on the curve and has coordinates (4, 1). A straight line L, through Q, is perpendicular to the tangent at Q.

(d) (i) Calculate the gradient of L.
(ii) Find the equation of L.
(iii) The line L intersects the curve again at R. Find the x-coordinate of R.

(8)

(Total 13 marks)

13. The parabola shown has equation $y^2 = 9x$.



(a) Verify that the point P (4, 6) is on the parabola.

(2)

The line (PQ) is the normal to the parabola at the point P, and cuts the x-axis at Q.

(b) (i) Find the equation of (PQ) in the form $ax + by + c = 0$.

(5)

(ii) Find the coordinates of Q.

(2)

S is the point $\left(\frac{9}{4}, 0\right)$.

(c) Verify that $SP = SQ$.

(4)

(d) The line (PM) is parallel to the x-axis. From part (c), explain why (QP) bisects the angle $\hat{SPM}$.

(3)

(Total 16 marks)

14. The function f is given by

$$f(x) = \frac{2x+1}{x-3},\ x \in \mathbb{R},\ x \neq 3.$$

(a) (i) Show that $y = 2$ is an asymptote of the graph of $y = f(x)$.

(2)

(ii) Find the vertical asymptote of the graph.

(1)

(iii) Write down the coordinates of the point P at which the asymptotes intersect.

(1)

(b) Find the points of intersection of the graph and the axes.

(4)

(c) Hence sketch the graph of $y = f(x)$, showing the asymptotes by dotted lines.

(4)

(d) Show that $f'(x) = \frac{-7}{(x-3)^2}$ and hence find the equation of the tangent at the point S where $x = 4$.

(6)

(e) The tangent at the point T on the graph is parallel to the tangent at S. Find the coordinates of T.

(5)

(f) Show that P is the midpoint of [ST].

(l)

(Total 24 marks)