CHAPTER 3 REVIEW SOLUTIONS – FOR SLs ONLY

1. $y = x^2 - x$

$\frac{dy}{dx} = 2x - 1$ = gradient at any point. (M1)

Line parallel to $y = 5x$

$\Rightarrow 2x - 1 = 5$ (M1)

$x = 3$ (A1)

$y = 6$ (A1)

Point (3, 6) (C2)(C2)

[4]

2. $y = x^3 + 1$

$\frac{dy}{dx} = 3x^2$

= Slope of tangent at any point

Therefore at point where $x = 1$, slope = 3 (M1)

$\Rightarrow$ Slope of normal $= -\frac{1}{3}$ (M1)(A1)

$\Rightarrow$ Equation of normal: $y - 2 = -\frac{1}{3}(x - 1)$

$3y - 6 = -x + 1$

$x + 3y - 7 = 0$ (A1) (C4)

Note: *Accept equivalent forms eg* $y = -\frac{1}{3}x + 2\frac{1}{3}$

[4]

3. (a) valid approach R1

e.g. $f''(x) = 0$, the max and min of f' gives the points of inflexion on f

–0.114, 0.364 (accept (–0.114, 0.811) and (0.364, 2.13)) A1A1 N1N1

(b) **METHOD 1**

graph of g is a quadratic function R1 N1

a quadratic function does not have any points of inflexion R1 N1

METHOD 2

graph of g is concave down over entire domain R1 N1

therefore no change in concavity R1 N1

METHOD 3

$g''(x) = -144$ R1 N1

therefore no points of inflexion as $g''(x) \neq 0$ R1 N1

[5]

4. **METHOD 1**

$l + 2w = 60$ (M1)

$l = 60 - 2w$ (A1)

$A = w(60 - 2w)$ $(= 60w - 2w^2)$ (A1)

$\frac{dA}{dw} = 60 - 4w$ (A1)

Using $\frac{dA}{dw} = 0$ $(60 - 4w = 0)$ (M1)

$w = 15$ (A1) (C6)

METHOD 2

$w + 2l = 60$ (A1)

$w = 60 - 2l$ (A1)

$A = l(60 - 2l)$ $(= 60l - 2l^2)$ (A1)

$\frac{dA}{dl} = 60 - 4l$ (A1)

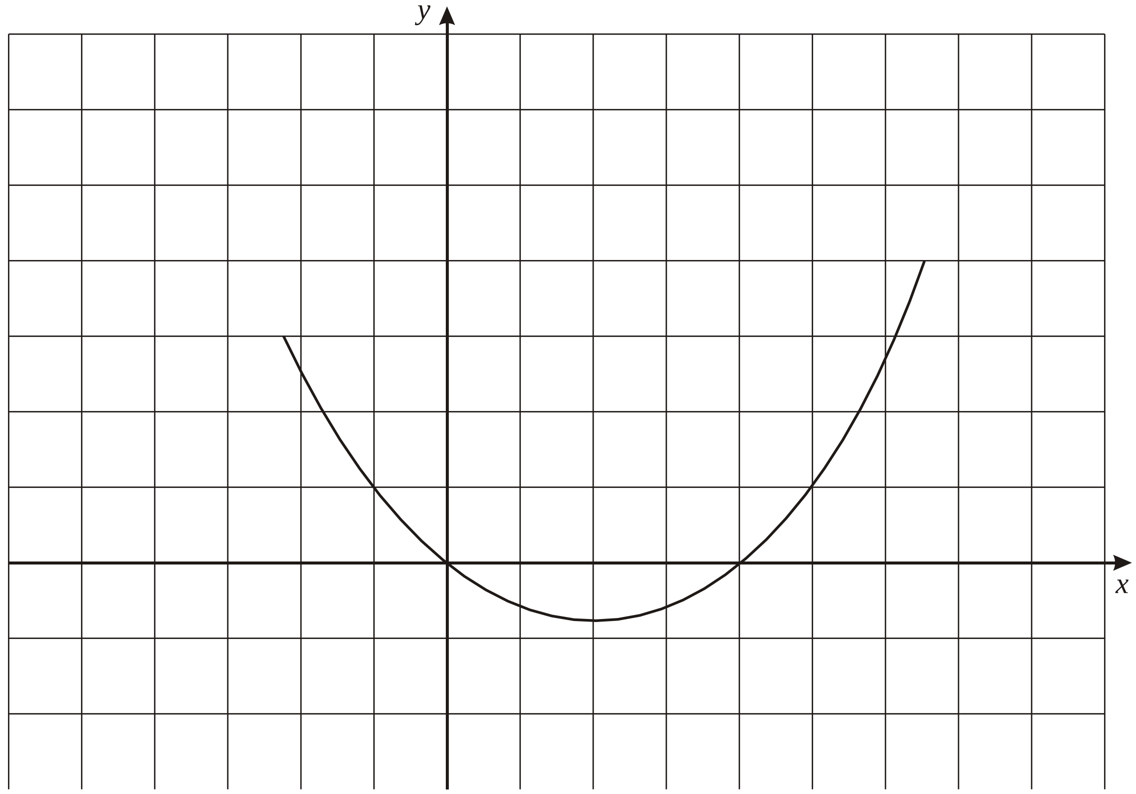
Using $\frac{dA}{dl} = 0$ $(60 - 4l = 0)$ (M1)

$l = 15$

$w = 30$ (A1) (C6)

[6]

5.

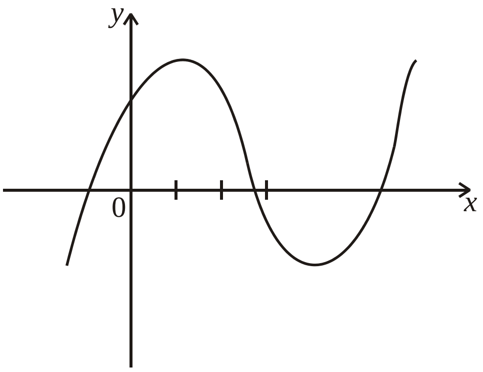


(A2)(A1)(A1)(A2) (C6)

Note: *Award A2 for correct shape (approximately parabolic), A1 A1 for intercepts at 0 and 4, A2 for minimum between x = 1.5 and x = 2.5.*

[6]

6. **METHOD 1**



Using gdc coordinates of maximum are
(0.667, 26.9) (G3)(G3) (C6)

METHOD 2

At maximum $\frac{dy}{dx} = 3x^2 - 20x + 12 = 0 = (3x - 2)(x - 6)$ (M1)(A1)(M1)

=> $x = \frac{2}{3}$ must be where maximum occurs (A1)

$x = \frac{2}{3} \Rightarrow y = \left(\frac{2}{3}\right)^3 - 10\left(\frac{2}{3}\right)^2 + 12\left(\frac{2}{3}\right) + 23 = \frac{725}{27}$ (= 26.9, 3 sf) (M1)(A1)

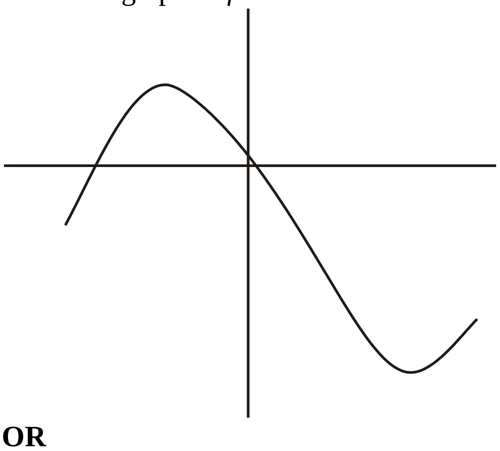
Maximum at $\left(\frac{2}{3}, \frac{725}{27}\right)$ (C4)(C2)

[6]

7. (a) **EITHER**

Recognizing that tangents parallel to the x-axis mean maximum and minimum (may be seen on sketch) R1

Sketch of graph of f M1



OR

Evidence of using $f'(x) = 0$ M1

Finding $f'(x) = 3x^2 - 6x - 24$ A1

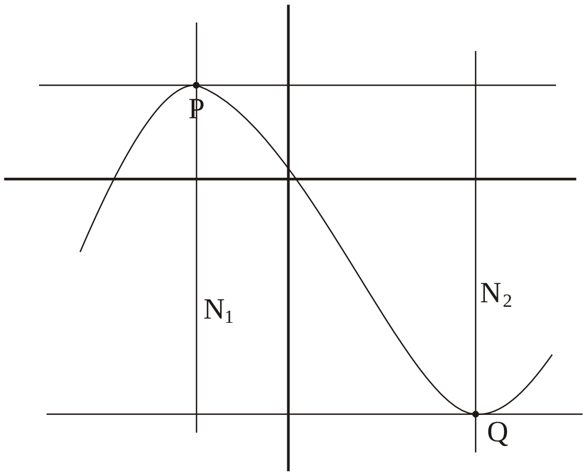
$3x^2 - 6x - 24 = 0$

Solutions $x = -2$ or $x = 4$

THEN

Coordinates are P(–2, 29) and Q(4, –79) A1A1 N1N1

(b)



(i)	(4, 29)	A1	N1
(ii)	(–2, –79)	A1	N1

[6]

8. $f'(x) = 12x^2 + 2$ A1A1

When $x = 1$, $f(1) = 6$ (seen anywhere) (A1)

When $x = 1$, $f'(1) = 14$ (A1)

Evidence of taking the negative reciprocal (M1)

eg $\frac{-1}{14}x$, $\frac{1}{-14}$, -0.0714

Equation is $y - 6 = -\frac{1}{14}(x-1)$ $\left(y = -\frac{1}{14}x + \frac{85}{14}, y = -0.0714x + 6.07\right)$ A1 N4

[6]

9. (a) (i) $f'(x) = 0$ A1 N1

(ii) **METHOD 1**

$f'(x) < 0$ to the left of C, $f'(x) > 0$ to the right of C R1R1 N2

METHOD 2

$f''(x) > 0$ R2 N2

(b) A A1 N1

(c) **METHOD 1**

$f''(x) = 0$ R2

Discussion of sign change of $f''(x)$ R1

e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B

B is a point of inflexion AG N0

METHOD 2

B is a minimum on the graph of the derivative f' R2

Discussion of sign change of $f''(x)$ R1

e.g. $f''(x) < 0$ to the left of B and $f''(x) > 0$ to the right of B; $f''(x)$ changes sign either side of B

B is a point of inflexion AG N0

[7]

10. **METHOD 1**

correct expression for **second** side, using area = 525 (A1)

e.g. let AB = x, AD = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side (M1)

e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $\frac{525}{x}\times 3+\frac{525}{x}\times 3+11x+3x, \frac{525}{\text{AB}}\times 3+\frac{525}{\text{AB}}\times 3+11\text{AB}+3\text{AB}, \frac{3150}{x}+14x$

EITHER

sketch of cost function (M1)

identifying minimum point (A1)

e.g. marking point on graph, x = 15

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = \frac{-1575}{x^2}+\frac{-1575}{x^2}+14$

setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $\frac{-3150}{x^2}+14=0$

minimum cost is 420 (dollars) A1 N4

METHOD 2

correct expression for **second** side, using area = 525 (A1)

e.g. let AD = x, AB = $\frac{525}{x}$

attempt to set up cost function using \$3 for three sides and \$11 for one side (M1)

e.g. 3(AD + BC + CD) + 11AB

correct expression for cost A2

e.g. $3\left(x+x+\frac{525}{x}\right)+\frac{525}{x}\times 11, 3\left(\text{AD}+\text{AD}+\frac{525}{\text{AD}}\right)+\frac{525}{\text{AD}}\times 11, 6x+\frac{7350}{x}$

EITHER

sketch of cost function (M1)

identifying minimum point (A1)

e.g. marking point on graph, x = 35

minimum cost is 420 (dollars) A1 N4

OR

correct derivative (may be seen in equation below) (A1)

e.g. $C'(x) = 6 - \frac{7350}{x^2}$

setting their derivative equal to 0 (seen anywhere) (M1)

e.g. $6 - \frac{7350}{x^2} = 0$

minimum cost is 420 (dollars) A1 N4

[7]

11. (a) Curve intersects y-axis when $x = 0$ (A1)

Gradient of tangent at y-intercept = 2 A1

$\Rightarrow$ gradient of $N = -\frac{1}{2}(= -0.5)$ A1

Finding y-intercept, 2.5 A1

Therefore, equation of N is $y = -0.5x + 2.5$ AG N0

(b) N intersects curve when $-0.5x^2 + 2x + 2.5 = -0.5x + 2.5$ A1

Solving equation (M1)

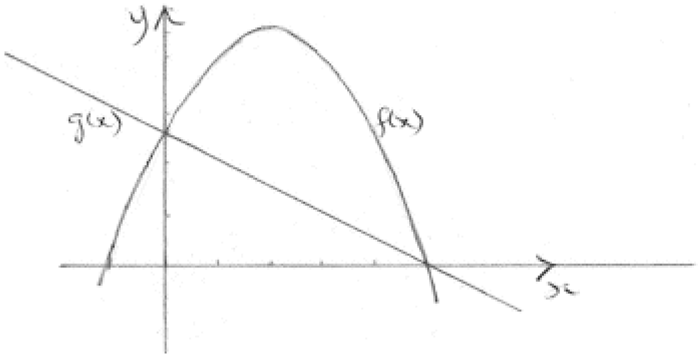
e.g. sketch, factorising

$\Rightarrow$ $x = 0$ or $x = 5$ A1

Other point when $x = 5$ (R1)

$x = 5 \Rightarrow y = 0$ (so other point (5, 0)) A1 N2

(c)



Using appropriate method, with subtraction/correct expression, **correct** limitsM1A1

e.g. $\int_0^5 f(x)\mathrm{d}x - \int_0^5 g(x)\mathrm{d}x, \int_0^5 (-0.5x^2 + 2.5x)\mathrm{d}x$

Area = 10.4 A2 N2

[13]

12. (a) $h = 3$ (A1)

$k = 2$ (A1) 2

(b) $f(x) = -(x-3)^2 + 2$

$= -x^2 + 6x - 9 + 2$ (must be a correct expression) (A1)

$= -x^2 + 6x - 7$ (AG) 1

(c) $f'(x) = -2x + 6$ (A2) 2

(d) (i) tangent gradient $= -2$ (A1)

gradient of $L = \frac{1}{2}$

(A1) (N2) 2

(ii) **EITHER**

equation of L is $y = \frac{1}{2}x + c$ (M1)

$c = -1.$ (A1)

$y = \frac{1}{2}x - 1$

OR

$y - 1 = \frac{1}{2}(x - 4)$

(A2) (N2) 2

(iii) **EITHER**

$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (M1)

$2x^2 - 11x + 12 = 0$ (may be implied) (A1)

$(2x - 3)(x - 4) = 0$ (may be implied) (A1)

$x = 1.5$

(A1) (N3) 4

OR

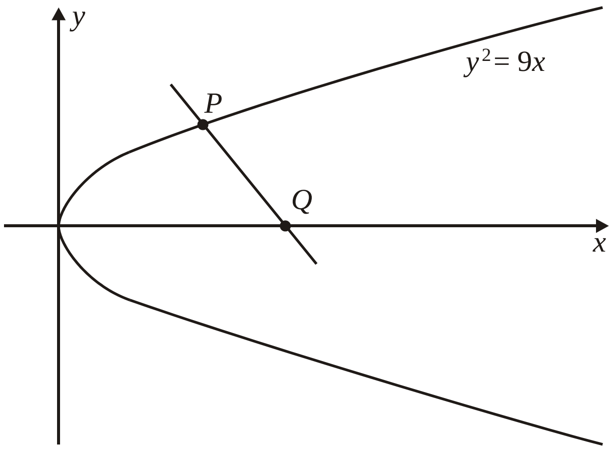
$-x^2 + 6x - 7 = \frac{1}{2}x - 1$ (or a sketch) (M1)

$x = 1.5$

(A3) (N3) 8

[13]

13. (a)



$y^2 = 9x$

$6^2 = 9(4)$ (M1)

$36 = 36$ (A1) 2

$\Rightarrow$ (4, 6) on parabola

(b) (i) $y = 3\sqrt{x}$

$\frac{dy}{dx} = \frac{3}{2\sqrt{x}}$ (M1)

= Slope at any point

Therefore at (4, 6), slope of tangent = $\frac{3}{4}$ (A1)

$\Rightarrow$ Slope of normal = $-\frac{4}{3}$ (A1)

Therefore equation of normal is $y - 6 = -\frac{4}{3}(x - 4)$ (M1)

$3y - 18 = -4x + 16$

$4x + 3y - 34 = 0$ (A1) 5

Notes: Candidates may differentiate implicitly to obtain $\frac{dy}{dx} = \frac{9}{2y}$.

Answer must be given in the form $ax + by + c = 0$.

(ii) Coordinates of Q:

$y = 0$, $4x = 34$

$x = \frac{17}{2}$ (A1)

$Q\left(\frac{17}{2}, 0\right)$ (A1) 2

(c) $SP = \sqrt{\left(\frac{9}{4} - 4\right)^2 + (0-6)^2}$ (M1)

$= \sqrt{\frac{49}{16} + 36}$

$= \frac{25}{4}$ (A1)

$SQ = \frac{17}{2} - \frac{9}{4}$ (M1)

$= \frac{34}{4} - \frac{9}{4}$

$= \frac{25}{4}$ (A1) 4

(d) $|SP| = |SQ| \Rightarrow S\hat{P}Q = S\hat{Q}P$ (M1)

But $S\hat{Q}P = M\hat{P}Q$ (alternate angles) (A1)

$\Rightarrow M\hat{P}Q = S\hat{P}Q$ (A1) 3

[16]

14. (a) (i) $f(x) = \frac{2x+1}{x-3}$

$= 2 + \frac{7}{x-3}$ by division or otherwise (M1)

Therefore as $|x| \to \infty$ $f(x) \to 2$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR $\lim_{x\to\infty} \frac{2x+1}{x-3} = 2$ (M1)(A1)

$\Rightarrow y = 2$ is an asymptote (AG)

OR make x the subject

$yx - 3y = 2x + 1$

$x(y-2) = 1 + 3y$ (M1)

$x = \frac{1+3y}{y-2}$ (A1)

$\Rightarrow y = 2$ is an asymptote (AG)

***Note:** Accept inexact methods based on the ratio of the coefficients of x.*

(ii) Asymptote at $x = 3$ (A1)

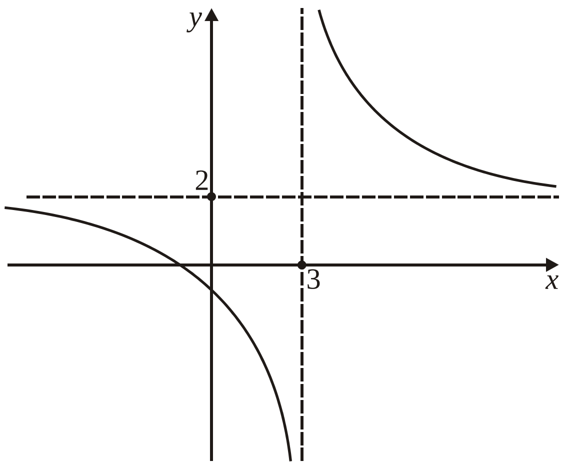
(iii) $P(3, 2)$ (A1) 4

(b) $f(x) = 0 \Rightarrow x = -\frac{1}{2}$ $\left(-\frac{1}{2}, 0\right)$ (M1)(A1)

$x = 0 \Rightarrow f(x) = -\frac{1}{3}$ $\left(0, -\frac{1}{3}\right)$ (M1)(A1) 4

***Note:** These do not have to be in coordinate form.*

(c)



(A4) 4

Note: *Asymptotes (A1)*
Intercepts (A1)
"Shape" (A2).

(d) $f'(x) = \dfrac{(x-3)(2)-(2x+1)}{(x-3)^2}$ (M1)

$= \dfrac{-7}{(x-3)^2}$ (A1)

= Slope at any point

Therefore slope when $x = 4$ is -7 (A1)

And $f(4) = 9$ ie $S(4, 9)$ (A1)

$\Rightarrow$ Equation of tangent: $y - 9 = -7(x - 4)$ (M1)

$7x + y - 37 = 0$ (A1) 6

(e) at T, $\dfrac{-7}{(x-3)^2} = -7$ (M1)

$\Rightarrow (x-3)^2 = 1$ (A1)

$x - 3 = \pm 1$ (A1)

$\left.\begin{array}{l} x = 4 \text{ or } 2 \\ y = 9 \text{ or } -5 \end{array}\right\} \begin{array}{l} S(4,9) \\ T(2,-5) \end{array}$ (A1)(A1) 5

(f) Midpoint $[ST] = \left(\dfrac{4+2}{2}, \dfrac{9-5}{2}\right)$

= (3, 2)

= point P (A1) 1

[24]