KINEMATICS - SOLUTIONS

1. (a) equation of line in graph $a = -\dfrac{25}{60}t + 15$ A1

$\left(a = -\dfrac{5}{12}t + 15\right)$

(b) $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{5}{12}t + 15$ (M1)

$v = -\dfrac{5}{24}t^2 + 15t + c$ (A1)

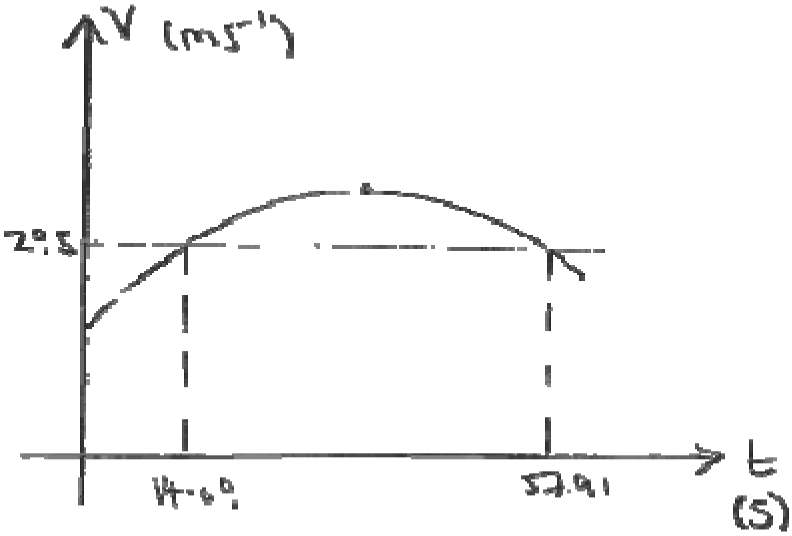
when $t = 0$, $v = 125$ m s^{-1}

$v = -\dfrac{5}{24}t^2 + 15t + 125$ A1

from graph or by finding time when $a = 0$

maximum = 395 m s^{-1} A1

(c) **EITHER**



graph drawn and intersection with $v = 295$ m s^{-1} (M1)(A1)

$t = 57.91 - 14.09 = 43.8$ A1

OR

$295 = -\dfrac{5}{24}t^2 + 15t + 125 \Rightarrow t = 57.91...; 14.09...$ (M1)(A1)

$t = 57.91\ldots - 14.09\ldots = 43.8$ ($8\sqrt{30}$) A1

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2. (a) $a = 10e^{-0.2t}$ (M1)(A1)

at $t = 10$, $a = 1.35$ (m s^{-2}) (accept $10e^{-2}$) A1

(b) **METHOD 1**

$d = \int_0^{10} 50(1 - e^{-0.2t})dt$ (M1)

$= 283.83...$ A1

so distance above ground = 1720 (m) (3 s.f.) (accept 1716 (m)) A1

METHOD 2

$s = \int 50(1 - e^{-0.2t})dt = 50t + 250e^{-0.2t}$ (+ c) M1

Taking $s = 0$ when $t = 0$ gives $c = -250$ M1

So when $t = 10$, $s = 283.3...$

so distance above ground = 1720 (m) (3 s.f.) (accept 1716 (m)) A1

[6]

3. (a) $\frac{dv}{dt} = -\frac{v^2}{200} - 32 \quad \left(= \frac{-v^2 - 6400}{200} \right)$ (M1)

$\int_0^T dt = \int_{40}^V -\frac{200}{v^2 + 80^2} dv$ M1A1A1

$T = 200 \int_V^{40} \frac{1}{v^2 + 80^2} dv$ AG

(b) (i) $a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt}$ R1

$= v\frac{dv}{ds}$ AG

(ii) $v\frac{dv}{ds} = \frac{-v^2 - 80^2}{200}$ (M1)

$\int_0^S ds = \int_{40}^V -\frac{200v}{v^2 + 80^2} dv$ M1A1A1

$\int_0^S ds = \int_V^{40} \frac{200v}{v^2 + 80^2} dv$ M1

$S = 200\int_V^{40} \frac{v}{v^2 + 80^2} dv$ A1

(c) letting $V = 0$ (M1)

distance = $200\int_0^{40} \frac{v}{v^2 + 80^2} dv$ = 22.3 metres A1

time = $200\int_0^{40} \frac{1}{v^2 + 80^2} dv$ = 1.16 seconds A1

[14]

4. $\dfrac{\mathrm{d}v}{\mathrm{d}t} = -\dfrac{1}{2}v$ A1

$\displaystyle\int \frac{\mathrm{d}v}{v} = \int -\frac{1}{2}\mathrm{d}t$ (A1)

$\ln v = -\dfrac{1}{2}t + c$ (A1)

$v = \mathrm{e}^{-\frac{1}{2}t+c} \quad \left(= A\mathrm{e}^{-\frac{1}{2}t}\right)$ (A1)

$t = 0$, $v = 40$, so $A = 40$ M1

$v = 40\mathrm{e}^{-\frac{1}{2}t}$ (or equivalent) A1

[6]

5. (a) $a = \dfrac{2s}{s^2+1}$

$a = v\dfrac{\mathrm{d}v}{\mathrm{d}s}$ M1

$v\dfrac{\mathrm{d}v}{\mathrm{d}s} = \dfrac{2s}{s^2+1}$

$\displaystyle\int v\mathrm{d}v = \int \frac{2s}{s^2+1}\mathrm{d}s$ M1

$\Rightarrow \dfrac{v^2}{2} = \ln\left|s^2+1\right| + k$ A1A1

Note: Do not penalize if k is missing.

When $s = 1$, $v = 2$

$\Rightarrow 2 = \ln 2 + k$ M1

$\Rightarrow k = 2 - \ln 2$ A1

$\Rightarrow \dfrac{v^2}{2} = \ln\left|s^2+1\right| + 2 - \ln 2 \quad \left(= \ln\left|\dfrac{s^2+1}{2}\right| + 2\right)$ A1

(b) **EITHER**

$\dfrac{v^2}{2} = \ln\left|\dfrac{26}{2}\right| + 2$ M1

$\Rightarrow v^2 = 2\ln\left|13\right| + 4$

$\Rightarrow v = \sqrt{2\ln\left|13\right| + 4}$ A1

OR

$\dfrac{v^2}{2} = \ln\left|26\right| + 2 - \ln 2$ M1

$v^2 = 2\ln\left|26\right| + 4 - 2\ln 2$

$v = \sqrt{2\ln\left|26\right| + 4 - 2\ln 2}$ A1

[9]