1. (a) $P(X < 30) = 0.4$
$P(X < 55) = 0.9$
or relevant sketch (M1)

given $Z = \dfrac{X-\mu}{\sigma}$

$P(Z < z) = 0.4 \Rightarrow \dfrac{30-\mu}{\sigma} = -0.253...$ (A1)

$P(Z < z) = 0.9 \Rightarrow \dfrac{55-\mu}{\sigma} = 1.28...$ (A1)

$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma$ M1

$\sigma = 16.3, \mu = 34.1$ A1

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two *M* marks.

(b) $X \sim N(34.12..., 16.28...^2)$
late to school $\Rightarrow X > 60$
$P(X > 60) = 0.056...$ (A1)
number of students late $= 0.0560... \times 1200$ (M1)
$= 67$ (to nearest integer) A1

Note: Accept 62 for use of 34 and 16.

(c) $P(X > 60 \mid X > 30) = \dfrac{P(X > 60)}{P(X > 30)}$ M1

$= 0.0935$ (accept anything between 0.093 and 0.094)A1

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

(d) let L be the random variable of the number of students who leave school in a 30 minute interval
since $24 \times 30 = 720$ A1
$L \sim Po(720)$
$P(L \geq 700) = 1 - P(L \leq 699)$ (M1)
$= 0.777$ A1

Note: Award M1A0 for $P(L > 700) = 1 - P(L \leq 700)$ (this leads to 0.765).

(e) (i) $Y \sim B(200, 0.7767\ldots)$ (M1)
$E(Y) = 200 \times 0.7767\ldots = 155$ A1

Note: On FT, use of 0.765 will lead to 153.

(ii) $P(Y > 150) = 1 - P(Y \le 150)$ (M1)

$= 0.797$ A1

Note: Accept 0.799 from using rounded answer.

Note: On FT, use of 0.765 will lead to 0.666.

[17]

2. (a) $P(x < 1.4) = 0.691$ (accept 0.692) A1

(b) **METHOD 1**

$Y \sim B(6, 0.3085...)$ (M1)

$P(Y \ge 4) = 1 - P(Y \le 3)$ (M1)

$= 0.0775$ (accept 0.0778 if 3 s.f. approximation from (a) used) A1

METHOD 2

$X \sim B(6, 0.6914...)$ (M1)

$P(X \le 2)$ (M1)

$= 0.0775$ (accept 0.0778 if 3 s.f. approximation from (a) used) A1

(c) $P(x < 1 \mid x < 1.4) = \dfrac{P(x<1)}{P(x<1.4)}$ M1

$= \dfrac{0.06680...}{0.6914...}$

$= 0.0966$ (accept 0.0967) A1

[6]

3. weight of glass $= X$

$X \sim N(160, \sigma^2)$

$P(X < 160 + 14) = P(X < 174) = 0.75$ (M1)(A1)

Note: $P(X < 160 - 14) = P(X < 146) = 0.25$ can also be used.

$P\left(Z < \dfrac{14}{\sigma}\right) = 0.75$ (M1)

$\dfrac{14}{\sigma} = 0.6745...$ (M1)(A1)

$\sigma = 20.8$ A1

[6]

4. (a) required to solve $P\left(Z < \frac{21-15}{\sigma}\right) = 0.8$ (M1)

$\frac{6}{\sigma} = 0.842...$ (or equivalent) (M1)

$\Rightarrow \sigma = 7.13$ (days) A1 N1

(b) P (survival after 21 days) = 0.337 (M1)A1

[5]

5. $X \sim N(\mu, \sigma^2)$

$P(X \le 5) = 0.670 \Leftrightarrow \frac{5-\mu}{\sigma} = 0.4399...$ M1A1

$P(X > 7) = 0.124 \Leftrightarrow \frac{7-\mu}{\sigma} = 1.155...$ A1

solve simultaneously

$\mu + 0.4399\sigma = 5$ and $\mu + 1.1552\sigma = 7$ M1

$\mu = 3.77$ (3 sf) A1 N3

the expected weight loss is 3.77 kg

Note: Award A0 for $\mu = 3.78$ (answer obtained due to early rounding).

[5]

6. (a) $H \sim N(166.5, 5^2)$

$P(H \ge 170) = 0.242...$ (M1)(A1)

$0.242... \times 63 = 15.2$ A1

so, approximately 15 students

(b) correct mean: 161.5 (cm) A1

variance remains the same, i.e. 25 (cm^2) A2

[6]

7. (a) $X \sim N(998, 2.5^2)$ M1

$P(X > 1000) = 0.212$ AG

(b) $X \sim B(5, 0.2119...)$

evidence of binomial (M1)

$P(X = 3) = \binom{5}{3}(0.2119...)^3 (0.7881...)^2 = 0.0591$ (accept 0.0592) (M1)A1

(c) $P(X \geq 1) = 1 - P(X = 0)$ (M1)

$1 - (0.7881...)^n > 0.99$

$(0.7881...)^n < 0.01$ A1

Note: Award A1 for line 2 or line 3 or equivalent.

$n > 19.3$ (A1)

minimum number of bottles required is 20 A1N2

(d) $\frac{996-\mu}{\sigma} = -1.1998$ (accept -1.2) M1A1

$\frac{1000-\mu}{\sigma} = 0.3999$ (accept 0.4) M1A1

$\mu = 999$ (ml), $\sigma = 2.50$ (ml) A1A1

(e) (i) $\frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!} + \frac{e^{-m}m^4}{4!}$ M1A1

$\frac{m^2}{2} = \frac{m^3}{6} + \frac{m^4}{24}$

$12m^2 - 4m^3 - m^4 = 0$ (A1)

$m = -6, 0, 2$

$\Rightarrow m = 2$ A1N2

(ii) $P(X > 2) = 1 - P(X \leq 2)$ (M1)

$= 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$= 1 - e^{-2} - 2e^{-2} - \frac{2^2 e^{-2}}{2!}$

$= 0.323$ A1

[20]

8. the waiting time, $X \sim N(18, 4^2)$

(a) $P(X > 25) = 0.0401$ (M1)A1

(b) $P\left(X < 20 \mid X > 15\right)$

$= \frac{P(15 < X < 20)}{P(X > 15)}$ (A1)

Note: Only one of the above A1 marks can be implied.

$= \frac{0.4648...}{0.7733...} = 0.601$ (M1)A1

[6]

9. (a) $P(X \leq 84) = P(Z \leq -1.62...) = 0.0524$ (M1)A1 N2

Note: Accept 0.0526.

(b) $P(Z \le z) = 0.01 \Rightarrow z = -2.326...$ (M1)

$P(X \le x) = P(Z \le z) = 0.01 \Rightarrow z = -2.326...$

$x = 81.4$ (accept 81) A1 N2

(c) $P(X \le 84) = 0.12 \Rightarrow z = -1.1749...$ (M1)

mean is 88.3 (accept 88) A1 N2

[6]

10. (a) (i) $P(4.8 < X < 7.5) = P(-0.8 < Z < 1)$ (M1)

$= 0.629$ A1 N2

Note: Accept $P(4.8 \le X \le 7.5) = P(-0.8 \le Z \le 1)$.

(ii) Stating $P(X < d) = 0.15$ **or** sketching an appropriately labelled diagram. A1

$\frac{d-6}{1.5} = -1.0364...$ (M1)(A1)

$d = (-1.0364...)(1.5) + 6$ (M1)

$= 4.45$ (km) A1 N4

(b) Stating **both** $P(X > 8) = 0.1$ and $P(X < 2) = 0.05$ **or** sketching an appropriately labelled diagram. R1

Setting up two equations in μ and σ (M1)

$8 = \mu + (1.281...)\sigma$ **and** $2 = \mu - (1.644...)\sigma$ A1

Attempting to solve for μ and σ (including by graphical means) (M1)

$\sigma = 2.05$ (km) **and** $\mu = 5.37$ (km) A1A1 N4

Note: Accept $\mu = 5.36, 5.38$.

(c) (i) Use of the Poisson distribution in an inequality. M1

$P(T \ge 3) = 1 - P(T \le 2)$ (A1)

$= 0.679...$ A1

Required probability is $(0.679...)^2 = 0.461$ M1A1 N3

Note: Allow FT for their value of $P(T \ge 3)$.

(ii) $\tau \sim \text{Po}(17.5)$ A1

$$P(\tau = 15) = \frac{e^{-17.5}(17.5)^{15}}{15!} \quad \text{(M1)}$$

$= 0.0849$ A1 N2

[21]

11. (a) $\bar{x} = \frac{412.11}{241} = 1.71$ A1

$$s^2 = \frac{705.5721}{240} - \frac{412.11^2}{240 \times 241} = 0.0036 \quad \text{M1A1}$$

(b) (i) H_0: Data can be modelled by a normal distribution
H_1: Data cannot be modelled by a normal distribution A1

(ii) The expected frequencies are

Interval	$x < 1.60$	$1.60 \le x < 1.65$	$1.65 \le x < 1.70$	$1.70 \le x < 1.75$	$1.75 \le x < 1.80$	$x \ge 1.80$
Exp Freq	8.04	30.19	66.31	75.60	44.75	16.10

A1A1A1A1A1A1

$$\chi^2 = \frac{5^2}{8.04} + \frac{34^2}{30.19} + \ldots + \frac{12^2}{16.10} - 241 = 3.30 / 3.29 \quad \text{M1A1}$$

Degrees of freedom = 3 A1
Critical value = 6.251 or p-value = 0.35 A1
The data can be modelled by a normal distribution. R1

[15]

12. (a) $X \sim N(231, 1.5^2)$
$P(X < 228) = 0.0228$ (M1)A1

Note: Accept 0.0227.

(b) (i) $X \sim N(\mu, 1.5^2)$
$P(X < 228) = 0.002$

$$\frac{228 - \mu}{1.5} = -2.878\ldots \quad \text{M1A1}$$

$\mu = 232$ grams A1 N3

(ii) $X \sim N(231, \sigma^2)$

$\frac{228-231}{\sigma} = -2.878...$ M1A1

$\sigma = 1.04$ grams A1 N3

(c) $X \sim B(100, 0.002)$ (M1)

$P(X \le 1) = 0.982...$ (A1)

$P(X \le 2) = 1 - P(X \le 1) = 0.0174$ A1

[11]

13. $P(X > 90) = 0.15$ **and** $P(X < 40) = 0.12$ (M1)

Finding standardized values 1.036, –1.175 A1A1

Setting up the equations $1.036 = \frac{90-\mu}{\sigma}, -1.175 = \frac{40-\mu}{\sigma}$ (M1)

$\mu = 66.6$, $\sigma = 22.6$ A1A1 N2N2

[6]