1. A student arrives at a school X minutes after 08:00, where X may be assumed to be normally distributed. On a particular day it is observed that 40 % of the students arrive before 08:30 and 90 % arrive before 08:55.

(a) Find the mean and standard deviation of X.

(5)

(b) The school has 1200 students and classes start at 09:00. Estimate the number of students who will be late on that day.

(3)

(c) Maelis had not arrived by 08:30. Find the probability that she arrived late.

(2)

At 15:00 it is the end of the school day, and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average, 24 students leave the school every minute.

(d) Find the probability that at least 700 students leave school before 15:30.

(3)

(e) There are 200 days in a school year. Given that Y denotes the number of days in the year that at least 700 students leave before 15:30, find

(i) $E(Y)$;

(ii) $P(Y > 150)$.

(4)
(Total 17 marks)

2. The fish in a lake have weights that are normally distributed with a mean of 1.3 kg and a standard deviation of 0.2 kg.

(a) Determine the probability that a fish that is caught weighs less than 1.4 kg.

(1)

(b) John catches 6 fish. Calculate the probability that at least 4 of the fish weigh more than 1.4 kg.

(3)

(c) Determine the probability that a fish that is caught weighs less than 1 kg, given that it weighs less than 1.4 kg.

(2)
(Total 6 marks)

3. In a factory producing glasses, the weights of glasses are known to have a mean of 160 grams. It is also known that the interquartile range of the weights of glasses is 28 grams. Assuming the weights of glasses to be normally distributed, find the standard deviation of the weights of glasses.

(Total 6 marks)

4. After being sprayed with a weedkiller, the survival time of weeds in a field is normally distributed with a mean of 15 days.

(a) If the probability of survival after 21 days is 0.2, find the standard deviation of the survival time.

(3)

When another field is sprayed, the survival time of weeds is normally distributed with a mean of 18 days.

(b) If the standard deviation of the survival time is unchanged, find the probability of survival after 21 days.

(2)
(Total 5 marks)

5. The weight loss, in kilograms, of people using the slimming regime *SLIM3M* for a period of three months is modelled by a random variable X. Experimental data showed that 67 % of the individuals using *SLIM3M* lost up to five kilograms and 12.4 % lost at least seven kilograms. Assuming that X follows a normal distribution, find the expected weight loss of a person who follows the *SLIM3M* regime for three months.

(Total 5 marks)

6. Bob measured the heights of 63 students. After analysis, he conjectured that the height, H, of the students could be modelled by a normal distribution with mean 166.5 cm and standard deviation 5 cm.

(a) Based on this assumption, estimate the number of these students whose height is at least 170 cm.

(3)

Later Bob noticed that the tape he had used to measure the heights was faulty as it started at the 5 cm mark and not at the zero mark.

(b) What are the correct values of the mean and variance of the distribution of the heights of these students?

(3)
(Total 6 marks)

7. Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(1)

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(3)

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water is greater than 0.99.

(4)

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(6)

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $P(X = 2) = P(X = 3) + P(X = 4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

(6)
(Total 20 marks)

8. Tim goes to a popular restaurant that does not take any reservations for tables. It has been determined that the waiting times for a table are normally distributed with a mean of 18 minutes and standard deviation of 4 minutes.

(a) Tim says he will leave if he is not seated at a table within 25 minutes of arriving at the restaurant. Find the probability that Tim will leave without being seated.

(2)

(b) Tim has been waiting for 15 minutes. Find the probability that he will be seated within the next five minutes.

(4)
(Total 6 marks)

9. A company produces computer microchips, which have a life expectancy that follows a normal distribution with a mean of 90 months and a standard deviation of 3.7 months.

(a) If a microchip is guaranteed for 84 months find the probability that it will fail before the guarantee ends.

(2)

(b) The probability that a microchip does not fail before the end of the guarantee is required to be 99%. For how many months should it be guaranteed?

(2)

(c) A rival company produces microchips where the probablity that they will fail after 84 months is 0.88. Given that the life expectancy also follows a normal distribution with standard deviation 3.7 months, find the mean.

(2)
(Total 6 marks)

10. The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.

(a) (i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15% of students travel less than d km to attend Gauss College. Find the value of d.

(7)

At Euler College, the distance travelled by students to attend their school is modelled by a normal distribution with mean μ km and standard deviation σ km.

(b) If 10% of students travel more than 8 km and 5% of students travel less than 2 km, find the value of μ and of σ.

(6)

The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5.

(c) (i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval.

(8)
(Total 21 marks)

11. The heights, x metres, of the 241 new entrants to a men's college were measured and the following statistics calculated.

$$\sum x = 412.11, \sum x^2 = 705.5721$$

(a) Calculate unbiased estimates of the population mean and the population variance.

(3)

(b) The Head of Mathematics decided to use a χ^2 test to determine whether or not these heights could be modelled by a normal distribution. He therefore divided the data into classes as follows.

Interval	$x < 1.60$	$1.60 \le x < 1.65$	$1.65 \le x < 1.70$	$1.70 \le x < 1.75$	$1.75 \le x < 1.80$	$x \ge 1.80$
Frequency	5	34	70	72	48	12

(i) State suitable hypotheses.

(ii) Calculate the value of the χ^2 statistic and state your conclusion using a 10% level of significance.

(12)
(Total 15 marks)

12. (a) A box of biscuits is considered to be underweight if it weighs less than 228 grams. It is known that the weights of these boxes of biscuits are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams. What is the probability that a box is underweight?

(2)

(b) The manufacturer decides that the probability of a box being underweight should be reduced to 0.002.

(i) Bill's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

(ii) Sarah's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.

(6)

(c) After the probability of a box being underweight has been reduced to 0.002, a group of customers buys 100 boxes of biscuits. Find the probability that at least two of the boxes are underweight.

(3)
(Total 11 marks)

13. The speeds of cars at a certain point on a straight road are normally distributed with mean μ and standard deviation σ. 15 % of the cars travelled at speeds greater than 90 km h^{-1} and 12 % of them at speeds less than 40 km h^{-1}. Find μ and σ.

(Total 6 marks)