1. (a) $P(X < 30) = 0.4$
$P(X < 55) = 0.9$
or relevant sketch (M1)

given $Z = \dfrac{X-\mu}{\sigma}$

$P(Z < z) = 0.4 \Rightarrow \dfrac{30-\mu}{\sigma} = -0.253...$ (A1)

$P(Z < z) = 0.9 \Rightarrow \dfrac{55-\mu}{\sigma} = 1.28...$ (A1)

$\mu = 30 + (0.253...) \times \sigma = 55 - (1.28...) \times \sigma$ M1
$\sigma = 16.3, \mu = 34.1$ A1

Note: Accept 16 and 34.

Note: Working with 830 and 855 will only gain the two *M* marks.

(b) $X \sim N(34.12..., 16.28...^2)$
late to school $\Rightarrow X > 60$
$P(X > 60) = 0.056...$ (A1)
number of students late = $0.0560... \times 1200$ (M1)
= 67 (to nearest integer) A1

Note: Accept 62 for use of 34 and 16.

(c) $P(X > 60 \mid X > 30) = \dfrac{P(X > 60)}{P(X > 30)}$ M1

= 0.0935 (accept anything between 0.093 and 0.094)A1

Note: If 34 and 16 are used 0.0870 is obtained. This should be accepted.

(d) let L be the random variable of the number of students who leave school in a 30 minute interval
since $24 \times 30 = 720$ A1
$L \sim Po(720)$
$P(L \geq 700) = 1 - P(L \leq 699)$ (M1)
$= 0.777$ A1

Note: Award M1A0 for $P(L > 700) = 1 - P(L \leq 700)$ (this leads to 0.765).

(e) (i) $Y \sim B(200, 0.7767\ldots)$ (M1)
$E(Y) = 200 \times 0.7767\ldots = 155$ A1

Note: On FT, use of 0.765 will lead to 153.

(ii) P(Y > 150) = 1 – P(Y ≤ 150) (M1)
= 0.797 A1

Note: Accept 0.799 from using rounded answer.

Note: On FT, use of 0.765 will lead to 0.666.

[17]

2. (a) $P(x = 0) = 0.607$ A1

(b) **EITHER**
Using $X \sim Po(3)$ (M1)

OR
Using $(0.6065...)^6$ (M1)

THEN
$P(X = 0) = 0.0498$ A1

(c) $X \sim Po(0.5t)$ (M1)
$P(x \geq 1) = 1 - P(x = 0)$ (M1)
$P(x = 0) < 0.01$ A1
$e^{-0.5t} < 0.01$ A1
$-0.5t < \ln(0.01)$ (M1)
$t > 9.21$ months
therefore 10 months A1N4

Note: Full marks can be awarded for answers obtained directly from GDC if a systematic method is used and clearly shown.

(d) (i) P(1 or 2 accidents) = 0.37908… A1
$E(B) = 1000 \times 0.60653... + 500 \times 0.37908...$ M1A1
= \$796 (accept \$797 or \$796.07) A1

(ii) P(2000) = P(1000, 1000, 0) + P(1000, 0, 1000)
+ P(0, 1000, 1000) + P(1000, 500, 500)
+ P(500, 1000, 500) + P(500, 500, 1000) (M1)(A1)

Note: Award M1 for noting that 2000 can be written both as $2 \times 1000 + 1 \times 0$ and $2 \times 500 + 1 \times 1000$.

$= 3(0.6065...)^2(0.01437...) + 3(0.3790...)^2(0.6065...)$ M1A1
= 0.277 (accept 0.278) A1

[18]

3. (a) number of patients in 30 minute period = X
$X \sim Po(3)$ (A1)
$P(X = 0) = 0.0498$ (M1)A1

(b) number of patients in working period = Y
$Y \sim \text{Po}(12)$ (A1)
$\text{P}(X < 10) = \text{P}(X \le 9) = 0.242$ (M1)A1

(c) number of working period with less than 10 patients = W
$W \sim \text{B}(6, 0.2424...)$ (M1)(A1)
$\text{P}(W \le 3) = 0.966$ (M1)A1

Note: Accept exact answers in parts (a) to (c).

(d) number of patients in t minute interval = X
$X \sim \text{Po}(T)$
$\text{P}(X \ge 2) = 0.95$
$\text{P}(X = 0) + \text{P}(X = 1) = 0.05$ (M1)(A1)
$\text{e}^{-T}(1 + T) = 0.05$ (M1)
$T = 4.74$ (A1)
$t = 47.4$ minutes A1

[15]

4. (a) required to solve $\text{e}^{-\lambda} + \lambda\text{e}^{-\lambda} = 0.123$ M1A1
solving to obtain $\lambda = 3.63$ A2 N2

Note: Award A2 if an additional negative solution is seen but A0 if only a negative solution is seen.

(b) $\text{P}(0 < X < 9)$
$= \text{P}(X \le 8) - \text{P}(X = 0)$ (or equivalent) (M1)
$= 0.961$ A1

[6]

5. (a) (i) mean = 6 (A1)

P(5 customers before 10:00) = $\frac{6^5}{5!}\text{e}^{-6} = 0.161$ (M1)A1

(ii) P(2 in 30 mins | 5 in 60 mins)

$= \frac{\text{P(2 in 30 mins)} \times \text{P(3 in next 30 mins)}}{\text{P(5 in 60 mins)}}$ (M1)(A1)

$= \frac{\frac{3^2}{2!}\text{e}^{-3} \times \frac{3^3}{3!}\text{e}^{-3}}{\frac{6^5}{5!}\text{e}^{-6}}$ A1

$= \frac{5}{16}$ (accept 0.312 or 0.313) A1

(b) (i) $\text{P}(T > t) = \text{P}$ 0 or 1 arrivals in $[0, t]$ = $\left(1 + \frac{t}{10}\right)\text{e}^{-\frac{t}{10}}$ R1

(ii) the distribution function is given by

$F(t) = 1 - \left(1+\frac{t}{10}\right)e^{-\frac{t}{10}}$ M1A1

the probability density is given, for $t > 0$, by

$f(t) = F'(t)$ (M1)

$= \frac{1}{10}\left(1+\frac{t}{10}\right)e^{-\frac{t}{10}} - \frac{1}{10}e^{-\frac{t}{10}}$ A1

$= \frac{t}{100}e^{-\frac{t}{10}}$ A1

(iii) $E(T) = \frac{1}{100}\int_0^\infty t^2 e^{-\frac{t}{10}}\,dt$ M1

$= -\frac{1}{10}\left[t^2 e^{-\frac{t}{10}}\right]_0^\infty + \frac{1}{10}\int_0^\infty 2te^{-\frac{t}{10}}\,dt$ M1A1

$= -2\left[te^{-\frac{t}{10}}\right]_0^\infty + 2\int_0^\infty e^{-\frac{t}{10}}\,dt$ M1A1

$= -20\left[e^{-\frac{t}{10}}\right]_0^\infty = 20$ A1

Note: Accept a method based on adding two exponential variables.

[19]

6. (a) $P(X = 1) + P(X = 3) = P(X = 0) + P(X = 2)$

$me^{-m} + \frac{m^3 e^{-m}}{6} = e^{-m} + \frac{m^2 e^{-m}}{2}$ (M1)(A1)

$m^3 - 3m^2 + 6m - 6 = 0$ (M1)

$m = 1.5961$ (4 decimal places) A1

(b) $m = 1.5961 \Rightarrow P(1 \le X \le 2) = me^{-m} + \frac{m^2 e^{-m}}{2} = 0.582$ (M1)A1

[6]

7. (a) $X \sim \text{Po}(0.6)$
$\text{P}(X \geq 1) = 1 - \text{P}(X = 0)$ M1
$= 0.451$ A1 N1

(b) $Y \sim \text{Po}(2.4)$ (M1)
$\text{P}(Y = 3) = 0.209$ A1

(c) $Z \sim \text{Po}(0.6n)$ (M1)
$\text{P}(Z \geq 3) = 1 - \text{P}(Z \leq 2) > 0.8$ (M1)

Note: Only one of these M1 marks may be implied.

$n \geq 7.132..$. (hours)
so, Mr Lee needs to fish for at least 8 complete hours A1 N2

Note: Accept a shown trial and error method that leads to a correct solution.

[7]

8. (a) $X \sim \text{N}(998, 2.5^2)$ M1
$\text{P}(X > 1000) = 0.212$ AG

(b) $X \sim \text{B}(5, 0.2119...)$
evidence of binomial (M1)
$\text{P}(X = 3) = \binom{5}{3}(0.2119...)^3\,(0.7881...)^2 = 0.0591$ (accept 0.0592) (M1)A1

(c) $\text{P}(X \geq 1) = 1 - \text{P}(X = 0)$ (M1)
$1 - (0.7881...)^n > 0.99$
$(0.7881...)^n < 0.01$ A1

Note: Award A1 for line 2 or line 3 or equivalent.

$n > 19.3$ (A1)
minimum number of bottles required is 20 A1N2

(d) $\frac{996-\mu}{\sigma} = -1.1998$ (accept -1.2) M1A1
$\frac{1000-\mu}{\sigma} = 0.3999$ (accept 0.4) M1A1
$\mu = 999$ (ml), $\sigma = 2.50$ (ml) A1A1

(e) (i) $\frac{e^{-m}m^2}{2!} = \frac{e^{-m}m^3}{3!} + \frac{e^{-m}m^4}{4!}$ M1A1

$\frac{m^2}{2} = \frac{m^3}{6} + \frac{m^4}{24}$

$12m^2 - 4m^3 - m^4 = 0$ (A1)

$m = -6, 0, 2$

$\Rightarrow m = 2$ A1N2

(ii) $P(X > 2) = 1 - P(X \le 2)$ (M1)

$= 1 - P(X = 0) - P(X = 1) - P(X = 2)$

$= 1 - e^{-2} - 2e^{-2} - \frac{2^2 e^{-2}}{2!}$

$= 0.323$ A1

[20]

9. (a) $P(X \le 2) = 0.4$ (M1)

$P(X = 0) + P(X = 1) + P(X = 2) = 0.4$ (A1)

$e^{-\lambda} + \lambda e^{-\lambda} + \frac{\lambda^2 e^{-\lambda}}{2!} = 0.4$ A1

mean, $\lambda = 3.11$ A1

(b) using a GDC (M1)

mode = 3 A1

[6]

10. (a) mean for 30 days: $30 \times 0.2 = 6$. (A1)

$P(X = 4) = \frac{6^4}{4!} e^{-6} = 0.134$ (M1)A1 N3

(b) $P(X > 3) = 1 - P(X \le 3) = 1 - e^{-6}(1 + 6 + 18 + 36) = 0.849$ (M1)A1 N2

(c) **EITHER**

mean for five days: $5 \times 0.2 = 1$ (A1)

$P(X = 0) = e^{-1}$ (= 0.368) A1 N2

OR

mean for one day: 0.2 (A1)

$P(X = 0) = (e^{-0.2})^5 = e^{-1}$ (= 0.368) A1 N2

(d) Required probability $= e^{-0.2} \times e^{-0.2} \times (1 - e^{-0.2})$ M1A1

$= 0.122$ A1 N3

(e) Expected cost is $1850 \times 6 = 11100$ euros A1

(f) On any one day $P(X = 0) = e^{-0.2}$

Therefore, $\binom{5}{1}\left(e^{-0.2}\right)^4\left(1-e^{-0.2}\right)=0.407$ M1A1 N2

[13]

11. (a) (i) $P(4.8 < X < 7.5) = P(-0.8 < Z < 1)$ (M1)

$= 0.629$ A1 N2

Note: Accept $P(4.8 \le X \le 7.5) = P(-0.8 \le Z \le 1)$.

(ii) Stating $P(X < d) = 0.15$ **or** sketching an appropriately labelled diagram. A1

$\frac{d-6}{1.5}=-1.0364...$ (M1)(A1)

$d = (-1.0364...)(1.5) + 6$ (M1)

$= 4.45$ (km) A1 N4

(b) Stating **both** $P(X > 8) = 0.1$ and $P(X < 2) = 0.05$ **or** sketching an appropriately labelled diagram. R1

Setting up two equations in μ and σ (M1)

$8 = \mu + (1.281...)\sigma$ **and** $2 = \mu - (1.644...)\sigma$ A1

Attempting to solve for μ and σ (including by graphical means) (M1)

$\sigma = 2.05$ (km) **and** $\mu = 5.37$ (km) A1A1 N4

Note: Accept $\mu = 5.36, 5.38$.

(c) (i) Use of the Poisson distribution in an inequality. M1

$P(T \ge 3) = 1 - P(T \le 2)$ (A1)

$= 0.679...$ A1

Required probability is $(0.679...)^2 = 0.461$ M1A1 N3

Note: Allow FT for their value of $P(T \ge 3)$.

(ii) $\tau \sim \text{Po}(17.5)$ A1

$$P(\tau = 15) = \frac{e^{-17.5}(17.5)^{15}}{15!}$$ (M1)

= 0.0849 A1 N2

[21]

12. Let X denote the number of imperfect glasses in the sample (M1)

For recognising binomial or proportion or Poisson A1
($X \sim B(200, p)$ where p-value is the probability of a glass being imperfect)
Let H_0: p-value = 0.02 and H_1: p-value > 0.02 A1A1

EITHER

p-value = 0.0493 A2
Using the binomial distribution p-value = 0.0493 > 0.01 we accept H_0 R1

OR

p-value = 0.0511 A2
Using the Poisson approximation to the binomial distribution since
p-value = 0.0511 > 0.01 we accept H_0 R1

OR

p-value = 0.0217 A2
Using the one proportion z-test since
p-value = 0.0217 > 0.01 we accept H_0 R1

Note: Use of critical values is acceptable.

[7]

13. (a) $$\text{Mean } \lambda = \frac{(9\times 0 + 12\times 1 + 22\times 2 + 10\times 3 + 11\times 4 + 8\times 5 + 8\times 6)}{80}$$ (M1)

$$= 2.725 \left(= \frac{109}{40}\right)$$ A1

Note: Do not accept 2.73.

(b) H_0: the data can be modelled by a Poisson distribution A1
H_1: the data cannot be modelled by a Poisson distribution A1

Number of calls	0	1	2	3	4	5	≥ 6
Observed frequency	9	12	22	10	11	8	8
Expected frequency	$\left(\frac{80\,e^{-2.725}(2.725)^0}{0!}\right)=5.244$	14.289	19.469	17.684	12.047	6.566	4.701

A3

Note: Award A2 for one error, A1 for two errors, A0 for three or more errors.

Combining last two columns (M1)

Note: Allow FT from not combining the last two columns and / or getting 2.98 for the last expected frequency.

EITHER

$$\chi^2=\frac{9^2}{5.244}+\frac{12^2}{14.289}+\frac{22^2}{19.469}+\frac{10^2}{17.684}+\frac{11^2}{12.047}+\frac{16^2}{11.267}-80$$ (M1)(A1)

= 8.804 (accept 8.8) A1

$v = 6 - 2 = 4$, $\chi^2_{5\%}=9.488$ A1A1

Hence 8.804 is not significant since 8.804 < 9.488 and we accept H_0 R1

OR

p-value = 0.0662 (accept 0.066) which is not significant since 0.0662 > 0.05 and we accept H_0 A5 R1 N0

[14]

14. (a) $$P(Z=n)=\sum_{k=0}^{n}e^{-\lambda}\times\frac{\lambda^k}{k!}\times e^{-\mu}\times\frac{\mu^{n-k}}{(n-k)!}$$ M1A1

$$=\frac{e^{-(\mu+\lambda)}}{n!}\sum_{k=0}^{n}\frac{n!}{k!(n-k)!}\lambda^k\,\mu^{n-k}$$ M1A1

$$=\frac{e^{-(\mu+\lambda)}}{n!}(\lambda+\mu)^n$$ A1

This shows that Z is Poisson distributed with mean $(\lambda+\mu)$. R1

(b) The result is (trivially) true for $n = 1$. A1

Assuming it to be true for $n = k$, i.e. $\sum_{r=1}^{k} U_r \sim \text{Po}(km)$ M1

Consider $\sum_{r=1}^{k+1} U_r = \sum_{r=1}^{k} U_r + U_{k+1}$ M1A1

which, using (a) is $\text{Po}(km + m)$ i.e. $\text{Po}([k + 1]m)$ A1

Hence proved by induction since true for $n = k \Rightarrow$ true for
$n = k + 1$ and we have shown true for $n = 1$. R1

[12]

15. (a) $X \sim \text{Po}(3.2)$

$\text{P}(X = 4) = \dfrac{e^{-3.2}\,3.2^4}{4!}$

$= 0.178$ A1

(b) (i) $\text{Var}(Y) = \text{E}(Y^2) - \text{E}^2(Y)$ (M1)

$m = 5.5 - m^2$ A1

$m = 1.90$ (or $m = -2.90$ which is invalid) A1

(ii) $Y \sim \text{Po}(1.90)$

$\text{P}(Y = 3) = \dfrac{e^{-1.90}\,1.90^3}{3!}$ (M1)

$= 0.171$ A1

(c) Required probability $= 0.171 \times 0.178 = 0.0304$ (accept 0.0305) (M1)A1

[8]

16. $X \sim \text{Po}(\mu)$

$\text{P}(X = 10) = 2\text{P}(X = 9)$ (M1)

$\dfrac{e^{-\mu}\mu^{10}}{10!} = \dfrac{2e^{-\mu}\mu^{9}}{9!}$ A1A1

$\mu = \dfrac{10! \times 2}{9!} = 10 \times 2 = 20$ A1

$\text{E}(X) = 20$ A1

[5]

17. (a) $P(3 \le X \le 5) = P(X \le 5) - P(X \le 2)$ (M1)

$= 0.547$ A1 N2

(b) $P(X \ge 3) = 1 - P(X \le 2)$ (M1)

$= 0.762$ A1 N2

(c) $P(3 \le X \le 5 \mid X \ge 3) = \dfrac{P(3 \le X \le 5)}{P(X \ge 3)} \left(= \dfrac{0.547}{0.762} \right)$ (M1)

$= 0.718$ A1 N2

[6]