1. A student arrives at a school X minutes after 08:00, where X may be assumed to be normally distributed. On a particular day it is observed that 40 % of the students arrive before 08:30 and 90 % arrive before 08:55.

(a) Find the mean and standard deviation of X.

(5)

(b) The school has 1200 students and classes start at 09:00. Estimate the number of students who will be late on that day.

(3)

(c) Maelis had not arrived by 08:30. Find the probability that she arrived late.

(2)

At 15:00 it is the end of the school day, and it is assumed that the departure of the students from school can be modelled by a Poisson distribution. On average, 24 students leave the school every minute.

(d) Find the probability that at least 700 students leave school before 15:30.

(3)

(e) There are 200 days in a school year. Given that Y denotes the number of days in the year that at least 700 students leave before 15:30, find

(i) $E(Y)$;

(ii) $P(Y > 150)$.

(4)
(Total 17 marks)

2. The number of accidents that occur at a large factory can be modelled by a Poisson distribution with a mean of 0.5 accidents per month.

(a) Find the probability that no accidents occur in a given month.

(1)

(b) Find the probability that no accidents occur in a given 6 month period.

(2)

(c) Find the length of time, in complete months, for which the probability that at least 1 accident occurs is greater than 0.99.

(6)

(d) To encourage safety the factory pays a bonus of $1000 into a fund for workers if no accidents occur in any given month, a bonus of $500 if 1 or 2 accidents occur and no bonus if more than 2 accidents occur in the month.

(i) Calculate the expected amount that the company will pay in bonuses each month.

(ii) Find the probability that in a given 3 month period the company pays a total of exactly $2000 in bonuses.

(9)
(Total 18 marks)

3. Casualties arrive at an accident unit with a mean rate of one every 10 minutes. Assume that the number of arrivals can be modelled by a Poisson distribution.

(a) Find the probability that there are no arrivals in a given half hour period.

(3)

(b) A nurse works for a two hour period. Find the probability that there are fewer than ten casualties during this period.

(3)

(c) Six nurses work consecutive two hour periods between 8am and 8pm.
Find the probability that no more than three nurses have to attend to less than ten casualties during their working period.

(4)

(d) Calculate the time interval during which there is a 95 % chance of there being at least two casualties.

(5)
(Total 15 marks)

4. The random variable X follows a Poisson distribution with mean λ.

(a) Find λ if $P(X = 0) + P(X = 1) = 0.123$.

(4)

(b) With this value of λ, find $P(0 < X < 9)$.

(2)
(Total 6 marks)

5. After a shop opens at 09:00 the number of customers arriving in any interval of duration t minutes follows a Poisson distribution with mean $\frac{t}{10}$.

(a) (i) Find the probability that exactly five customers arrive before 10:00.

(ii) Given that exactly five customers arrive before 10:00, find the probability that exactly two customers arrive before 09:30.

(7)

(b) Let the second customer arrive at T minutes after 09:00.

(i) Show that, for $t > 0$,

$$P(T > t) = \left(1 + \frac{t}{10}\right)e^{-\frac{t}{10}}.$$

(ii) Hence find in simplified form the probability density function of T.

(iii) Evaluate $E(T)$.
(You may assume that, for $n \in \mathbb{Z}^+$ and $a > 0$, $\lim_{t \to \infty} t^n e^{-at} = 0$.)

(12)
(Total 19 marks)

6. The random variable X follows a Poisson distribution with mean m and satisfies

$$P(X = 1) + P(X = 3) = P(X = 0) + P(X = 2).$$

(a) Find the value of m correct to four decimal places.

(4)

(b) For this value of m, calculate $P(1 \le X \le 2)$.

(2)
(Total 6 marks)

7. Mr Lee is planning to go fishing this weekend. Assuming that the number of fish caught per hour follows a Poisson distribution with mean 0.6, find

(a) the probability that he catches at least one fish in the first hour;

(2)

(b) the probability that he catches exactly three fish if he fishes for four hours;

(2)

(c) the number of **complete** hours that Mr Lee needs to fish so that the probability of catching more than two fish exceeds 80 %.

(3)
(Total 7 marks)

8. Testing has shown that the volume of drink in a bottle of mineral water filled by **Machine A** at a bottling plant is normally distributed with a mean of 998 ml and a standard deviation of 2.5 ml.

(a) Show that the probability that a randomly selected bottle filled by Machine A contains more than 1000 ml of mineral water is 0.212.

(1)

(b) A random sample of 5 bottles is taken from Machine A. Find the probability that exactly 3 of them each contain more than 1000 ml of mineral water.

(3)

(c) Find the minimum number of bottles that would need to be sampled to ensure that the probability of getting at least one bottle filled by Machine A containing more than 1000 ml of mineral water is greater than 0.99.

(4)

(d) It has been found that for **Machine B** the probability of a bottle containing less than 996 ml of mineral water is 0.1151. The probability of a bottle containing more than 1000 ml is 0.3446. Find the mean and standard deviation for the volume of mineral water contained in bottles filled by Machine B.

(6)

(e) The company that makes the mineral water receives, on average, m phone calls every 10 minutes. The number of phone calls, X, follows a Poisson distribution such that $P(X = 2) = P(X = 3) + P(X = 4)$.

(i) Find the value of m.

(ii) Find the probability that the company receives more than two telephone calls in a randomly selected 10 minute period.

(6)
(Total 20 marks)

9. The random variable X has a Poisson distribution. Given that $P(X > 2) = 0.6$, find

(a) the mean of the distribution;

(4)

(b) the mode of the distribution.

(2)
(Total 6 marks)

10. The lifts in the office buildings of a small city have occasional breakdowns. The breakdowns at any given time are independent of one another and can be modelled using a Poisson Distribution with mean 0.2 per day.

(a) Determine the probability that there will be exactly four breakdowns during the month of June (June has 30 days).

(3)

(b) Determine the probability that there are more than three breakdowns during the month of June.

(2)

(c) Determine the probability that there are no breakdowns during the first five days of June.

(2)

(d) Find the probability that the first breakdown in June occurs on June 3^{rd}.

(3)

(e) It costs 1850 euros to service the lifts when they have breakdowns. Find the expected cost of servicing lifts for the month of June.

(1)

(f) Determine the probability that there will be no breakdowns in exactly 4 out of the first 5 days in June.

(2)
(Total 13 marks)

11. The distance travelled by students to attend Gauss College is modelled by a normal distribution with mean 6 km and standard deviation 1.5 km.

(a) (i) Find the probability that the distance travelled to Gauss College by a randomly selected student is between 4.8 km and 7.5 km.

(ii) 15% of students travel less than d km to attend Gauss College. Find the value of d.

(7)

At Euler College, the distance travelled by students to attend their school is modelled by a normal distribution with mean μ km and standard deviation σ km.

(b) If 10% of students travel more than 8 km and 5% of students travel less than 2 km, find the value of μ and of σ.

(6)

The number of telephone calls, T, received by Euler College each minute can be modelled by a Poisson distribution with a mean of 3.5.

(c) (i) Find the probability that at least three telephone calls are received by Euler College in **each** of two successive one-minute intervals.

(ii) Find the probability that Euler College receives 15 telephone calls during a randomly selected five-minute interval.

(8)
(Total 21 marks)

12. A factory makes wine glasses. The manager claims that on average 2% of the glasses are imperfect. A random sample of 200 glasses is taken and 8 of these are found to be imperfect.

Test the manager's claim at a 1% level of significance using a one-tailed test.

(Total 7 marks)

13. The number of telephone calls received by a helpline over 80 one-minute periods are summarized in the table below.

Number of calls	0	1	2	3	4	5	6
Frequency	9	12	22	10	11	8	8

(a) Find the exact value of the mean of this distribution.

(2)

(b) Test, at the 5% level of significance, whether or not the data can be modelled by a Poisson distribution.

(12)
(Total 14 marks)

14. (a) The independent random variables X and Y have Poisson distributions and $Z = X + Y$. The means of X and Y are λ and μ respectively. By using the identity

$$\mathrm{P}(Z = n) = \sum_{k=0}^{n} \mathrm{P}(X = k)\,\mathrm{P}(Y = n - k)$$

show that Z has a Poisson distribution with mean $(\lambda + \mu)$.

(6)

(b) Given that $U_1, U_2, U_3, \ldots$ are independent Poisson random variables each having mean m, use mathematical induction together with the result in (a) to show that $\sum_{r=1}^{n} U_r$ has a Poisson distribution with mean nm.

(6)
(Total 12 marks)

15. (a) Ahmed is typing Section A of a mathematics examination paper. The number of mistakes that he makes, X, can be modelled by a Poisson distribution with mean 3.2. Find the probability that Ahmed makes exactly four mistakes.

(1)

(b) His colleague, Levi, is typing Section B of this paper. The number of mistakes that he makes, Y, can be modelled by a Poisson distribution with mean m.

(i) If $\mathrm{E}(Y^2) = 5.5$, find the value of m.

(ii) Find the probability that Levi makes exactly three mistakes.

(5)

(c) Given that X and Y are independent, find the probability that Ahmed makes exactly four mistakes and Levi makes exactly three mistakes.

(2)
(Total 8 marks)

16. Flowering plants are randomly distributed around a field according to a Poisson distribution with mean μ. Students find that they are twice as likely to find exactly ten flowering plants as to find exactly nine flowering plants in a square metre of field. Calculate the expected number of flowering plants in a square metre of field.

(Total 5 marks)

17. The random variable X has a Poisson distribution with mean 4. Calculate

(a) $P(3 \le X \le 5)$;

(2)

(b) $P(X \ge 3)$;

(2)

(c) $P(3 \le X \le 5 | X \ge 3)$.

(2)

(Total 6 marks)