1. An open glass is created by rotating the curve $y = x^2$, defined in the domain $x \in [0, 10]$, 2π radians about the y-axis. Units on the coordinate axes are defined to be in centimetres.

(a) When the glass contains water to a height h cm, find the volume V of water in terms of h.

(3)

(b) If the water in the glass evaporates at the rate of 3 cm^3 per hour for each cm^2 of exposed surface area of the water, show that,

$$\frac{dV}{dt} = -3\sqrt{2\pi V}\text{ , where } t \text{ is measured in hours.}$$

(6)

(c) If the glass is filled completely, how long will it take for all the water to evaporate?

(7)
(Total 16 marks)

2. A rocket is rising vertically at a speed of 300 m s^{-1} when it is 800 m directly above the launch site. Calculate the rate of change of the distance between the rocket and an observer, who is 600 m from the launch site and on the same horizontal level as the launch site.

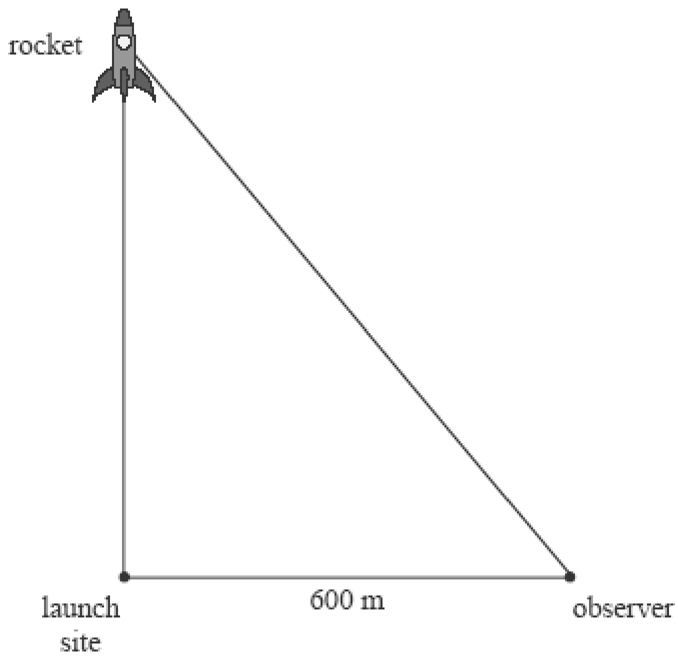
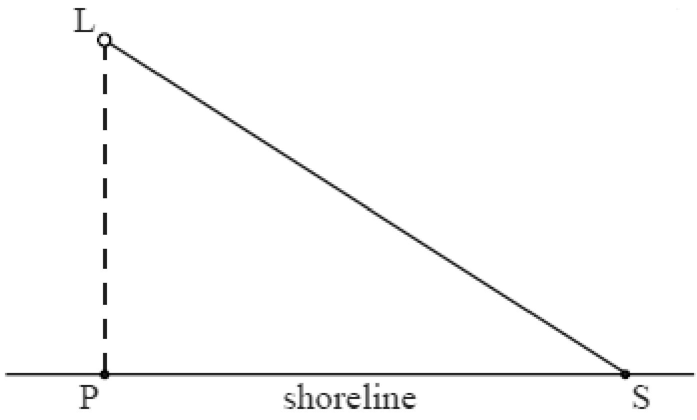


diagram not to scale

(Total 6 marks)

3. A lighthouse L is located offshore, 500 metres from the nearest point P on a long straight shoreline. The narrow beam of light from the lighthouse rotates at a constant rate of 8π radians per minute, producing an illuminated spot S that moves along the shoreline. You may assume that the height of the lighthouse can be ignored and that the beam of light lies in the horizontal plane defined by sea level.



When S is 2000 metres from P,

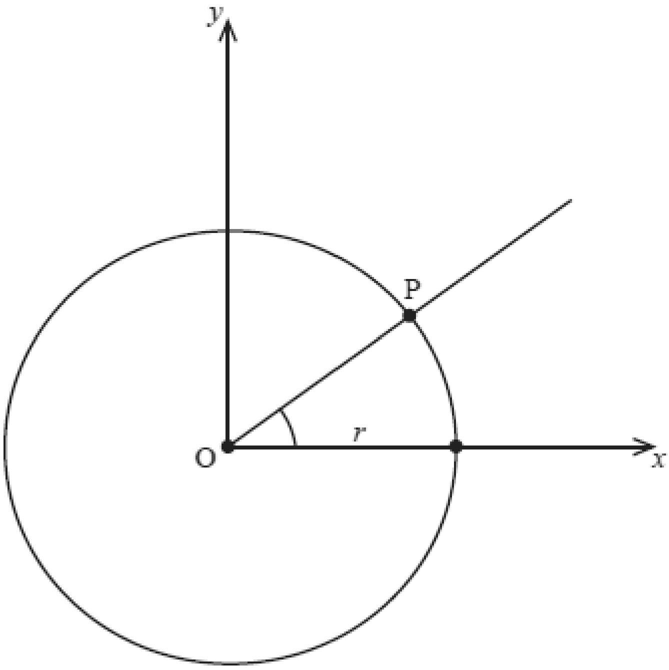
(a) show that the speed of S, correct to three significant figures, is 214 000 metres per minute;

(5)

(b) find the acceleration of S.

(3)
(Total 8 marks)

4. The diagram below shows a circle with centre at the origin O and radius $r > 0$.



A point P(x, y), ($x > 0$, $y > 0$) is moving round the circumference of the circle.

Let $m = \tan\left(\arcsin \frac{y}{r}\right)$.

(a) Given that $\frac{\mathrm{d}y}{\mathrm{d}t} = 0.001r$, show that $\frac{\mathrm{d}m}{\mathrm{d}t} = \left(\frac{r}{10\sqrt{r^2 - y^2}}\right)^3$.

(6)

(b) State the geometrical meaning of $\frac{\mathrm{d}m}{\mathrm{d}t}$.

(1)
(Total 7 marks)

5. A helicopter H is moving vertically upwards with a speed of 10 m s^{-1}. The helicopter is h m directly above the point Q, which is situated on level ground. The helicopter is observed from the point P, which is also at ground level, and PQ = 40 m.
This information is represented in the diagram below.

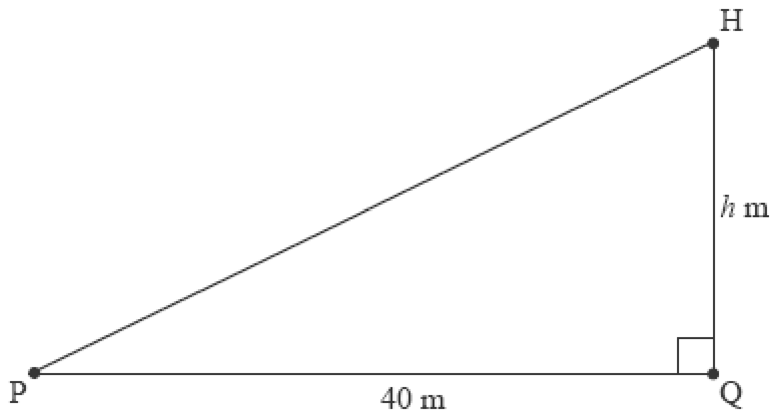


diagram not to scale

When $h = 30$,

(a) show that the rate of change of $\text{H}\hat{\text{P}}\text{Q}$ is 0.16 radians per second;

(3)

(b) find the rate of change of PH.

(4)
(Total 7 marks)

6. A gourmet chef is renowned for her spherical shaped soufflé. Once it is put in the oven, its volume increases at a rate proportional to its radius.

(a) Show that the radius r cm of the soufflé, at time t minutes after it has been put in the oven, satisfies the differential equation $\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{k}{r}$, where k is a constant.

(5)

(b) Given that the radius of the soufflé is 8 cm when it goes in the oven, and 12 cm when it's cooked 30 minutes later, find, to the nearest cm, its radius after 15 minutes in the oven.

(8)
(Total 13 marks)