

PRACTICE

Prove the following:

1. $\frac{1}{1+\cos\theta} - \frac{1}{1-\cos\theta} = -2\cot\theta\csc\theta$ 2. $\frac{1}{\tan\theta + \cot\theta} = \sin\theta\cos\theta$ 3. $\sec^2\theta + \csc^2\theta = \sec^2\theta\csc^2\theta$

4. $\frac{1+\sin\theta}{\cos\theta} = \sec\theta + \tan\theta$

5. $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$

Solve for all values:

6. $4\sin^2 x = 3$

7. $\tan^2 x = \tan x$

8. $\sec^2 x = 2\tan x$

9. $\csc^2 x + 2\cot x = 0$

10. $\cot^2 x + 2 = 2\csc x$

11. Let $f(x) = 3\sin x + 4\cos x$, for $-2\pi \leq x \leq 2\pi$.

(a) Sketch the graph of f .

(3)

(b) Write down

(i) the amplitude;

(ii) the period;

(iii) the x -intercept that lies between $-\frac{\pi}{2}$ and 0.

(3)

(c) Hence write $f(x)$ in the form $p \sin(qx + r)$.

(3)

(d) Write down the two values of k for which the equation $f(x) = k$ has exactly two solutions.

(2)

(Total 11 marks)

12. Let $f(x) = 5 \cos \frac{\pi}{4} x$ and $g(x) = -0.5x^2 + 5x - 8$, for $0 \leq x \leq 9$.

(a) On the same diagram, sketch the graphs of f and g .

(3)

(b) Consider the graph of f . Write down

(i) the x -intercept that lies between $x = 0$ and $x = 3$;

(ii) the period;

(iii) the amplitude.

(4)

(c) Consider the graph of g . Write down

(i) the two x -intercepts;

(ii) the equation of the axis of symmetry.

(3)

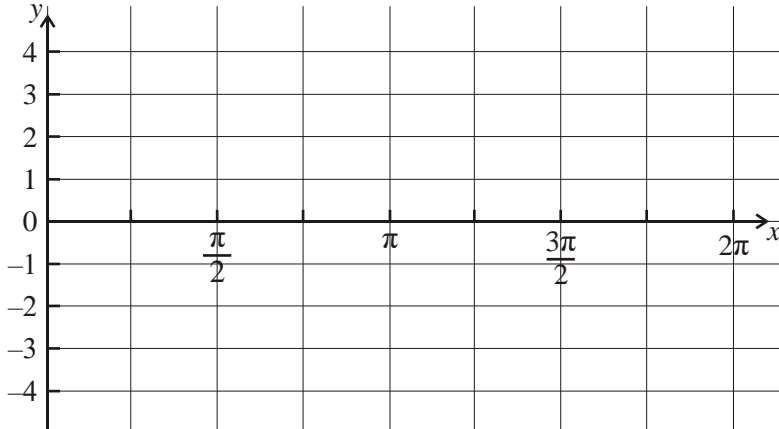
(Total 10 marks)

13. Consider $g(x) = 3 \sin 2x$.

(a) Write down the period of g .

(1)

(b) On the diagram below, sketch the curve of g , for $0 \leq x \leq 2\pi$.



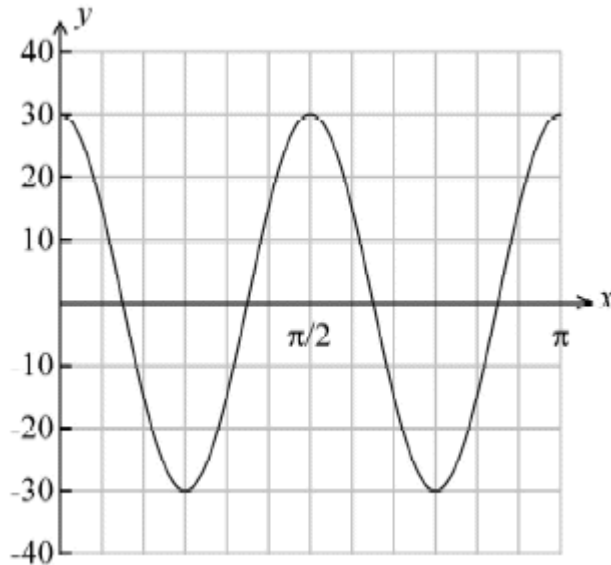
(3)

(c) Write down the number of solutions to the equation $g(x) = 2$, for $0 \leq x \leq 2\pi$.

(2)

(Total 6 marks)

14. The graph of a function of the form $y = p \cos qx$ is given in the diagram below.



(a) Write down the value of p .

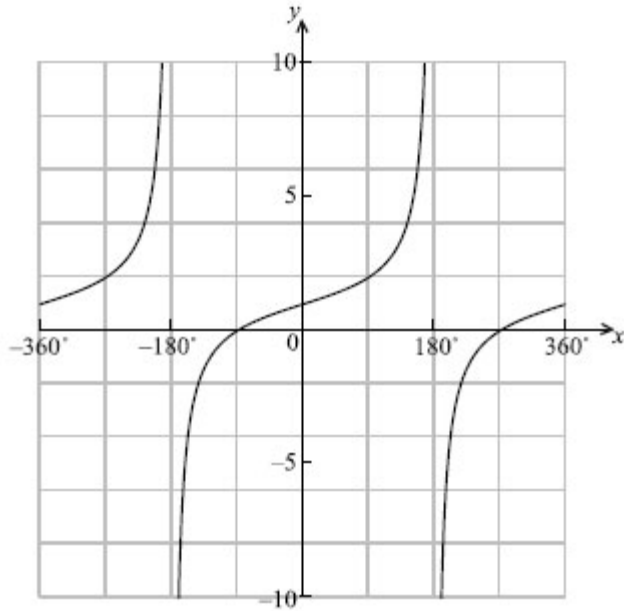
(2)

(b) Calculate the value of q .

(4)

(Total 6 marks)

15. The diagram below shows the graph of $f(x) = 1 + \tan\left(\frac{x}{2}\right)$ for $-360^\circ \leq x \leq 360^\circ$.



- (a) On the same diagram, draw the asymptotes.

(2)

- (b) Write down
 (i) the period of the function;
 (ii) the value of $f(90^\circ)$.

(2)

- (c) Solve $f(x) = 0$ for $-360^\circ \leq x \leq 360^\circ$.

(2)

(Total 6 marks)