# Chapter 15
## RULES OF DIFFERENTIATION

### EXERCISE 15A

**1** **a** $f(x) = x^3$
$\therefore\ f'(x) = 3x^2$

**b** $f(x) = 2x^3$
$\therefore\ f'(x) = 2(3x^2)$
$= 6x^2$

**c** $f(x) = 7x^2$
$\therefore\ f'(x) = 7(2x)$
$= 14x$

**d** $f(x) = 6\sqrt{x} = 6x^{\frac{1}{2}}$
$\therefore\ f'(x) = 6\left(\frac{1}{2}x^{-\frac{1}{2}}\right)$
$= \dfrac{3}{\sqrt{x}}$

**e** $f(x) = 3\sqrt[3]{x} = 3x^{\frac{1}{3}}$
$\therefore\ f'(x) = 3\left(\frac{1}{3}x^{-\frac{2}{3}}\right)$
$= \dfrac{1}{\sqrt[3]{x^2}}$

**f** $f(x) = x^2 + x$
$\therefore\ f'(x) = 2x + 1$

**g** $f(x) = 4 - 2x^2$
$\therefore\ f'(x) = 0 - 2(2x)$
$= -4x$

**h** $f(x) = x^2 + 3x - 5$
$\therefore\ f'(x) = 2x + 3 - 0$
$= 2x + 3$

**i** $f(x) = \frac{1}{2}x^4 - 6x^2$
$\therefore\ f'(x) = \frac{1}{2}(4x^3) - 6(2x)$
$= 2x^3 - 12x$

**j** $f(x) = \dfrac{3x-6}{x} = 3 - 6x^{-1}$
$\therefore\ f'(x) = 0 - 6(-1x^{-2})$
$= \dfrac{6}{x^2}$

**k** $f(x) = \dfrac{2x-3}{x^2} = \dfrac{2x}{x^2} - \dfrac{3}{x^2}$
$= 2x^{-1} - 3x^{-2}$
$\therefore\ f'(x) = -2x^{-2} + 6x^{-3} = -\dfrac{2}{x^2} + \dfrac{6}{x^3}$

**l** $f(x) = \dfrac{x^3+5}{x} = x^2 + 5x^{-1}$
$\therefore\ f'(x) = 2x - 5x^{-2}$
$= 2x - \dfrac{5}{x^2}$

**m** $f(x) = \dfrac{x^3+x-3}{x}$
$= x^2 + 1 - 3x^{-1}$
$\therefore\ f'(x) = 2x + 0 + 3x^{-2}$
$= 2x + \dfrac{3}{x^2}$

**n** $f(x) = \dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$
$\therefore\ f'(x) = -\frac{1}{2}x^{-\frac{3}{2}} = -\dfrac{1}{2x\sqrt{x}}$

**o** $f(x) = (2x-1)^2 = 4x^2 - 4x + 1$
$\therefore\ f'(x) = 8x - 4$

**p** $f(x) = (x+2)^3$
$= x^3 + 3x^2(2) + 3x(2^2) + 2^3$
$= x^3 + 6x^2 + 12x + 8$
$\therefore\ f'(x) = 3x^2 + 12x + 12$

**2** **a** $y = 2.5x^3 - 1.4x^2 - 1.3$
$\therefore\ \dfrac{dy}{dx} = 7.5x^2 - 2.8x$

**b** $y = \pi x^2$
$\therefore\ \dfrac{dy}{dx} = 2\pi x$

**c** $y = \dfrac{1}{5x^2} = \frac{1}{5}x^{-2}$
$\therefore\ \dfrac{dy}{dx} = -\frac{2}{5}x^{-3} = -\dfrac{2}{5x^3}$

**d** $y = 100x$
$\therefore\ \dfrac{dy}{dx} = 100$

**e** $y = 10(x+1)$
$= 10x + 10$
$\therefore\ \dfrac{dy}{dx} = 10$

**f** $y = 4\pi x^3$
$\therefore\ \dfrac{dy}{dx} = 12\pi x^2$

**3** **a** $\frac{d}{dx}(6x+2)$
$= 6$

**b** $\frac{d}{dx}(x\sqrt{x})$
$= \frac{d}{dx}(x^{\frac{3}{2}})$
$= \frac{3}{2}x^{\frac{1}{2}}$
$= \frac{3\sqrt{x}}{2}$

**c** $\frac{d}{dx}(5-x)^2$
$= \frac{d}{dx}(25-10x+x^2)$
$= -10+2x$
$= 2x-10$

**d** $\frac{d}{dx}\left(\frac{6x^2-9x^4}{3x}\right)$
$= \frac{d}{dx}(2x-3x^3)$
$= 2-9x^2$

**e** $\frac{d}{dx}((x+1)(x-2))$
$= \frac{d}{dx}(x^2-x-2)$
$= 2x-1$

**f** $\frac{d}{dx}\left(\frac{1}{x^2}+6\sqrt{x}\right)$
$= \frac{d}{dx}\left(x^{-2}+6x^{\frac{1}{2}}\right)$
$= -2x^{-3}+3x^{-\frac{1}{2}}$
$= -\frac{2}{x^3}+\frac{3}{\sqrt{x}}$

**g** $\frac{d}{dx}\left(4x-\frac{1}{4x}\right)$
$= \frac{d}{dx}\left(4x-\frac{1}{4}x^{-1}\right)$
$= 4+\frac{1}{4}x^{-2}$
$= 4+\frac{1}{4x^2}$

**h** $\frac{d}{dx}(x(x+1)(2x-5))$
$= \frac{d}{dx}\left(x(2x^2-3x-5)\right)$
$= \frac{d}{dx}\left(2x^3-3x^2-5x\right)$
$= 6x^2-6x-5$

**4** **a** Consider $y = x^2$ when $x = 2$
Now $\frac{dy}{dx} = 2x$
$\therefore$ when $x = 2$,
$\frac{dy}{dx} = 2(2) = 4$
$\therefore$ the tangent has gradient 4.

**b** Consider $y = \frac{8}{x^2}$ at the point $(9, \frac{8}{81})$
Now $y = 8x^{-2}$
$\therefore$ $\frac{dy}{dx} = -16x^{-3} = -\frac{16}{x^3}$
$\therefore$ at $(9, \frac{8}{81})$, $x = 9$ and so $\frac{dy}{dx} = -\frac{16}{729}$
$\therefore$ the tangent has gradient $-\frac{16}{729}$.

**c** Consider $y = 2x^2-3x+7$ when $x = -1$
Now $\frac{dy}{dx} = 4x-3$
$\therefore$ when $x = -1$,
$\frac{dy}{dx} = 4(-1)-3 = -7$
$\therefore$ the tangent has gradient $-7$.

**d** Consider $y = \frac{2x^2-5}{x}$ at the point $(2, \frac{3}{2})$
Now $y = 2x-5x^{-1}$
$\therefore$ $\frac{dy}{dx} = 2+5x^{-2} = 2+\frac{5}{x^2}$
$\therefore$ at $(2, \frac{3}{2})$, $x = 2$ and so $\frac{dy}{dx} = 2+\frac{5}{4}$
$= \frac{13}{4}$
$\therefore$ the tangent has gradient $\frac{13}{4}$.

**e** Consider $y = \frac{x^2-4}{x^2}$ at the point $(4, \frac{3}{4})$
Now $y = 1-4x^{-2}$
$\therefore$ $\frac{dy}{dx} = 0+8x^{-3} = \frac{8}{x^3}$
$\therefore$ at $(4, \frac{3}{4})$, $x = 4$ and so
$\frac{dy}{dx} = \frac{8}{4^3} = \frac{1}{8}$
$\therefore$ the tangent has gradient $\frac{1}{8}$.

**f** Consider $y = \frac{x^3-4x-8}{x^2}$ when $x = -1$
Now $y = x-4x^{-1}-8x^{-2}$
$\therefore$ $\frac{dy}{dx} = 1+4x^{-2}+16x^{-3}$
$= 1+\frac{4}{x^2}+\frac{16}{x^3}$
$\therefore$ when $x = -1$,
$\frac{dy}{dx} = 1+4-16 = -11$
$\therefore$ the tangent has gradient $-11$.

**5** $f(x) = x^2 + (b+1)x + 2c$, $f(2) = 4$, and $f'(-1) = 2$

$\therefore\ f'(x) = 2x + (b+1)$

But $f'(-1) = 2$, so $2(-1) + b + 1 = 2$

$\therefore\ -1 + b = 2$

$\therefore\ b = 3$

So, $f(x) = x^2 + (3+1)x + 2c$

$= x^2 + 4x + 2c$

But $f(2) = 4$, so $2^2 + 4(2) + 2c = 4$

$\therefore\ 2c = -8$

$\therefore\ c = -4$

**6** **a** $f(x) = 4\sqrt{x} + x = 4x^{\frac{1}{2}} + x$

$\therefore\ f'(x) = 4\left(\frac{1}{2}x^{-\frac{1}{2}}\right) + 1$

$= \dfrac{2}{\sqrt{x}} + 1$

**b** $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$

$\therefore\ f'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$= \dfrac{1}{3\sqrt[3]{x^2}}$

**c** $f(x) = -\dfrac{2}{\sqrt{x}} = -2x^{-\frac{1}{2}}$

$\therefore\ f'(x) = -2\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$

$= x^{-\frac{3}{2}}$

$= \dfrac{1}{x\sqrt{x}}$

**d** $f(x) = 2x - \sqrt{x} = 2x - x^{\frac{1}{2}}$

$\therefore\ f'(x) = 2 - \frac{1}{2}x^{-\frac{1}{2}}$

$= 2 - \dfrac{1}{2\sqrt{x}}$

**e** $f(x) = \dfrac{4}{\sqrt{x}} - 5 = 4x^{-\frac{1}{2}} - 5$

$\therefore\ f'(x) = 4\left(-\frac{1}{2}x^{-\frac{3}{2}}\right)$

$= -2x^{-\frac{3}{2}} = -\dfrac{2}{x\sqrt{x}}$

**f** $f(x) = 3x^2 - x\sqrt{x} = 3x^2 - x^{\frac{3}{2}}$

$\therefore\ f'(x) = 6x - \frac{3}{2}x^{\frac{1}{2}}$

$= 6x - \frac{3}{2}\sqrt{x}$

**g** $f(x) = \dfrac{5}{x^2\sqrt{x}} = 5x^{-\frac{5}{2}}$

$\therefore\ f'(x) = 5\left(-\frac{5}{2}x^{-\frac{7}{2}}\right)$

$= -\frac{25}{2}x^{-\frac{7}{2}}$

$= \dfrac{-25}{2x^3\sqrt{x}}$

**h** $f(x) = 2x - \dfrac{3}{x\sqrt{x}} = 2x - 3x^{-\frac{3}{2}}$

$\therefore\ f'(x) = 2 - 3\left(-\frac{3}{2}x^{-\frac{5}{2}}\right)$

$= 2 + \frac{9}{2}x^{-\frac{5}{2}}$

$= 2 + \dfrac{9}{2x^2\sqrt{x}}$

**7** **a** $y = 4x - \dfrac{3}{x} = 4x - 3x^{-1}$ $\therefore\ \dfrac{dy}{dx} = 4 + 3x^{-2} = 4 + \dfrac{3}{x^2}$

$\dfrac{dy}{dx}$ is the gradient function of $y = 4x - \dfrac{3}{x}$ from which the gradient at any point can be found.

**b** $S = 2t^2 + 4t$ m $\therefore\ \dfrac{dS}{dt} = 4t + 4\ \text{m s}^{-1}$

$\dfrac{dS}{dt}$ is the instantaneous rate of change in position at time $t$. It is the velocity function.

**c** $C = 1785 + 3x + 0.002x^2$ dollars.

$\dfrac{dC}{dx} = 3 + 0.002(2x) = 3 + 0.004x$ dollars per toaster

$\dfrac{dC}{dx}$ is the instantaneous rate of change in cost as the number of toasters changes.