Math SL May 10 NOTES: applications of derivatives - kinematics & optimization

- If the signs of $v(t)$ and $a(t)$ are the same (both positive or both negative), then the speed of P is increasing.
- If the signs of $v(t)$ and $a(t)$ are opposite, then the speed of P is decreasing.

A particle P moves along the x-axis with position given by $x(t) = 1 - 2\cos t$ cm where t is the time in seconds.

a State the initial position, velocity and acceleration of P.

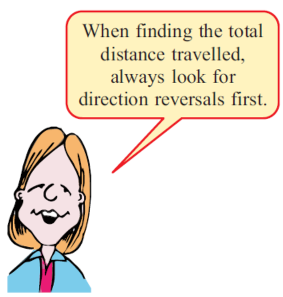
b Describe the motion when $t = \frac{\pi}{4}$ seconds.

c Find the times when the particle reverses direction on $0 < t < 2\pi$ and find the position of the particle at these instants.

d When is the particle's speed increasing on $0 \leqslant t \leqslant 2\pi$?

The position of a particle moving along the x-axis is given by $x(t) = t^3 - 9t^2 + 24t$ metres where t is in seconds, $t \geqslant 0$.

a Draw sign diagrams for the particle's velocity and acceleration functions.



b Find the position of the particle at the times when it reverses direction, and hence draw a motion diagram for the particle.

c At what times is the particle's:

i speed decreasing **ii** velocity decreasing?

d Find the total distance travelled by the particle in the first 5 seconds of motion.

OPTIMISATION PROBLEM SOLVING METHOD

Step 1: Draw a large, clear diagram of the situation.

Step 2: Construct a formula with the variable to be **optimised** as the subject. It should be written in terms of **one** convenient **variable**, for example x. You should write down what domain restrictions there are on x.

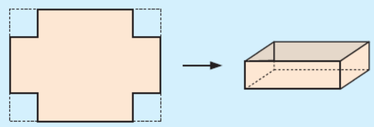
Step 3: Find the **first derivative** and find the values of x which make the first derivative **zero**.

Step 4: For a restricted domain such as $a \leqslant x \leqslant b$, the maximum or minimum may occur either when the derivative is zero, at an endpoint, or at a point where the derivative is not defined. Show using the **sign diagram test**, the **second derivative test**, or the **graphical test**, that you have a maximum or a minimum.

Step 5: Write your answer in a sentence, making sure you specifically answer the question.

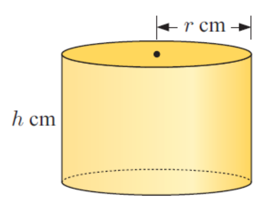
A rectangular cake dish is made by cutting out squares from the corners of a 25 cm by 40 cm rectangle of tin-plate, and then folding the metal to form the container.

What size squares must be cut out to produce the cake dish of maximum volume?



Consider the manufacture of cylindrical tin cans of 1 L capacity where the cost of the metal used is to be minimised. This means that the surface area must be as small as possible.

a Explain why the height h is given by $h = \dfrac{1000}{\pi r^2}$ cm.



b Show that the total surface area A is given by $A = 2\pi r^2 + \dfrac{2000}{r}$ cm^2.

c Use *technology* to help you sketch the graph of A against r.

d Find the value of r which makes A as small as possible.

e Sketch the can of smallest surface area.