Friday Oct. 19 Textbook HW Solutions:

REVIEW SET 5A

1 $f(x) = x^2 - 2x$

a $f(3) = 3^2 - 2(3) = 9 - 6 = 3$

b $f(2x) = (2x)^2 - 2(2x) = 4x^2 - 4x$

c $f(-x) = (-x)^2 - 2(-x) = x^2 + 2x$

d $3f(x) - 2 = 3(x^2 - 2x) - 2 = 3x^2 - 6x - 2$

2 $f(x) = 5 - x - x^2$

a $f(-1) = 5 - (-1) - (-1)^2 = 5 + 1 - 1 = 5$

b $f(x-1) = 5 - (x-1) - (x-1)^2 = 5 - x + 1 - [x^2 - 2x + 1] = 6 - x - x^2 + 2x - 1 = -x^2 + x + 5$

c $f\left(\frac{x}{2}\right) = 5 - \left(\frac{x}{2}\right) - \left(\frac{x}{2}\right)^2 = 5 - \frac{1}{2}x - \frac{1}{4}x^2$

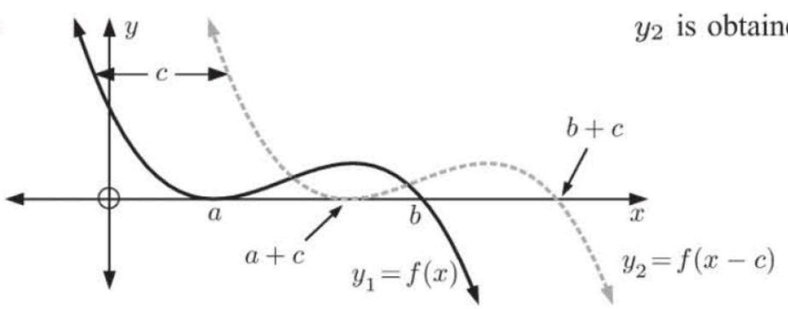
d $2f(x) - f(-x) = 2(5 - x - x^2) - \left[5 - (-x) - (-x)^2\right] = 10 - 2x - 2x^2 - [5 + x - x^2] = 10 - 2x - 2x^2 - 5 - x + x^2 = -x^2 - 3x + 5$

3 $f(x) = 3x^3 - 2x^2 + x + 2$

If $g(x)$ is $f(x)$ translated $\binom{1}{-2}$, then

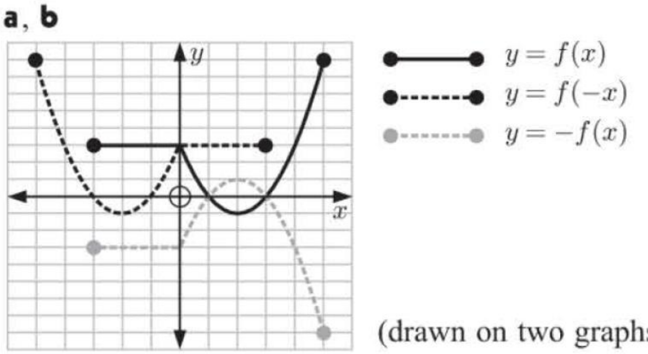
$$\begin{aligned} g(x) &= f(x-1) - 2 \\ &= 3(x-1)^3 - 2(x-1)^2 + (x-1) + 2 - 2 \\ &= 3(x^3 - 3x^2 + 3x - 1) - 2(x^2 - 2x + 1) + x - 1 \\ &= 3x^3 - 9x^2 + 9x - 3 - 2x^2 + 4x - 2 + x - 1 \\ &= 3x^3 - 11x^2 + 14x - 6 \end{aligned}$$

4



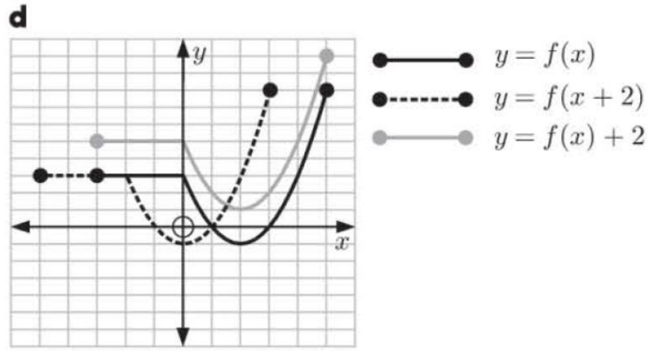
y_2 is obtained by translating y_1 c units to the right.

5 **a, b**



(drawn on two graphs)

c, d



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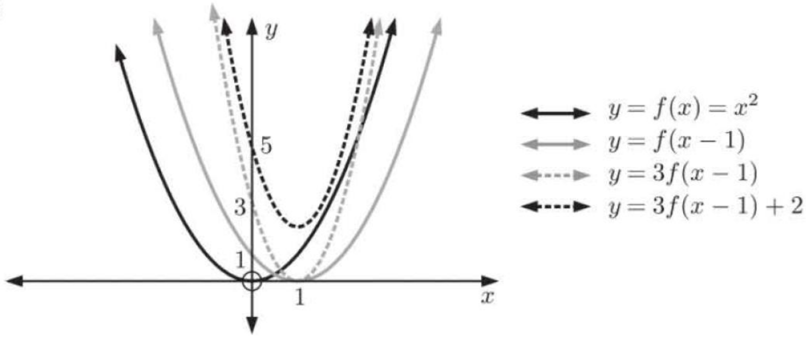
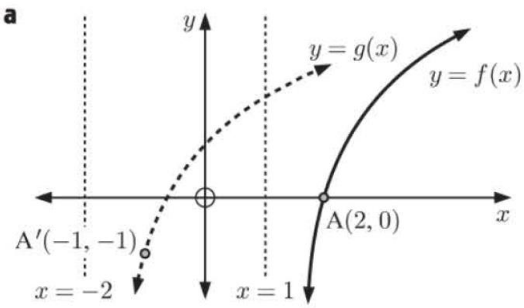


7 **a**



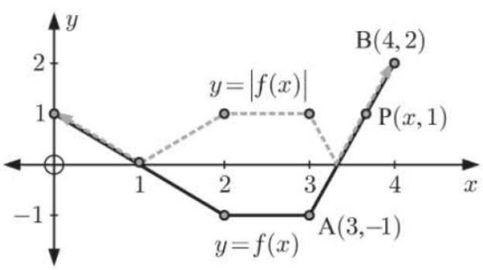
b $f(x+3)-1$ is a translation of $f(x)$ by $\binom{-3}{-1}$.

$\therefore$ vertical asymptote is at $x = 1 - 3 = -2$.

c A(2, 0) translated by $\binom{-3}{-1}$ gives

$(2-3, 0-1)$ which is A′$(-1, -1)$.

8 **a**



b When $x = 0$,

$$\frac{1}{f(x)} = \frac{1}{f(0)} = \frac{1}{1} = 1$$

$\therefore$ the y-intercept of $\dfrac{1}{f(x)}$ is 1.

c Invariant points for $\dfrac{1}{f(x)}$ occur when $f(x) = \pm 1$.

$f(x) = -1$ for all $x \in [2, 3]$

$f(x) = 1$ when $x = 0$ and at point P.

To find the point P where $f(x) = 1$,

note that the gradient of $[\text{AB}] = \dfrac{2-(-1)}{4-3} = 3$,

so $\dfrac{2-1}{4-x} = 3$

$\therefore \quad 1 = 12 - 3x$

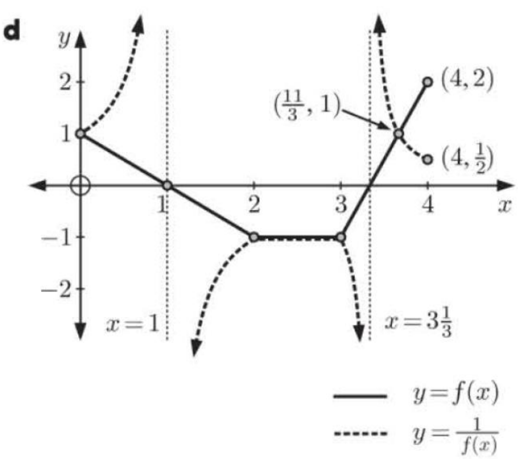
$\therefore \quad 3x = 11$

$\therefore \quad x = \frac{11}{3}$

$\therefore$ $f(x)$ is invariant for $\dfrac{1}{f(x)}$ at $(0, 1)$, $(\frac{11}{3}, 1)$,

and all the points on $y = -1$, $x \in [2, 3]$.

d



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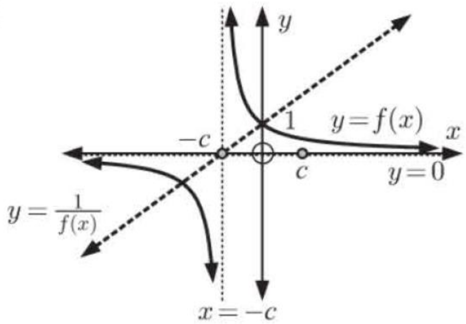


a $f(x) = \dfrac{c}{x+c}$ has a VA $x = -c$ $\{f(x)$ is undefined$\}$

and a HA $y = 0$ $\{$as $|x| \to \infty,\ f(x) \to 0\}$

$f(0) = \dfrac{c}{0+c} = 1$ $\therefore$ the y-intercept is 1

There are no x-intercepts $\{\dfrac{c}{x+c} \neq 0$ for $c > 0\}$

b $\dfrac{1}{f(x)} = \dfrac{x+c}{c}$

$\dfrac{1}{f(0)} = \dfrac{0+c}{c} = 1$ $\therefore$ the y-intercept of $\dfrac{1}{f(x)}$ is 1

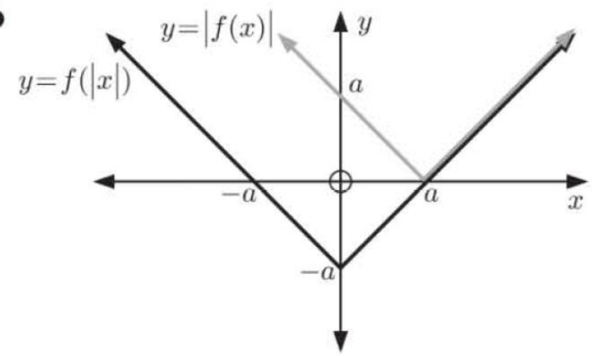
$\dfrac{1}{f(x)} = 0$ when $x + c = 0$ or $x = -c$

$\therefore$ the x-intercept of $\dfrac{1}{f(x)}$ is $-c$.

10 **a** $f(x) = x - a,\quad a > 0$

$\therefore\ |f(x)| = |x-a|$ and $f(|x|) = |x| - a$

b



c Using the graph in **b**, $|x-a| = |x| - a$ when $x \geqslant a$.

or solving algebraically:

For $x < 0$ and $a > 0$, $|x-a| = a - x$ and $|x| = -x$

If $|x-a| = |x| - a$ then $a - x = -x - a$

$\therefore\ 2a = 0$

$\therefore\ a = 0$ which is not true.

For $0 \leqslant x < a$ and $a > 0$, $|x-a| = a - x$ and $|x| = x$

If $|x-a| = |x| - a$ then $a - x = x - a$

$\therefore\ 2x = 2a$

$\therefore\ x = a$ which is not true.

For $x \geqslant a$ and $a > 0$, $|x-a| = x - a$ and $|x| = x$

If $|x-a| = |x| - a$ then $x - a = x - a$ which is true.

So, for $a > 0$, $|x-a| = |x| - a$ is true for all $x \geqslant a$.