1. (a) $1 - \mathrm{i}\sqrt{3}$ A1

(b) **EITHER**

$(z - (1 + \mathrm{i}\sqrt{3}))(z - (1 - \mathrm{i}\sqrt{3})) = z^2 - 2z + 4$ (M1)A1

$p(z) = (z - 2)(z^2 - 2z + 4)$ (M1)

$= z^3 - 4z^2 + 8z - 8$ A1

therefore $b = -4$, $c = 8$, $d = -8$

OR

relating coefficients of cubic equations to roots

$-b = 2 + 1 + \mathrm{i}\sqrt{3} + 1 - \mathrm{i}\sqrt{3} = 4$ M1

$c = 2(1 + \mathrm{i}\sqrt{3}) + 2(1 - \mathrm{i}\sqrt{3}) + (1 + \mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = 8$

$-d = 2(1 + \mathrm{i}\sqrt{3})(1 - \mathrm{i}\sqrt{3}) = 8$

$b = -4$, $c = 8$, $d = -8$ A1A1A1

(c) $z_2 = 2\mathrm{e}^{\frac{\mathrm{i}\pi}{3}}$, $z_3 = 2\mathrm{e}^{-\frac{\mathrm{i}\pi}{3}}$ A1A1A1

Note: Award A1 for modulus,
A1 for each argument.

[8]

2. $\mathrm{i}z_1 + 2z_2 = 3 \Rightarrow z_2 = -\frac{1}{2}\mathrm{i}z_1 + \frac{3}{2}$

$z_1 + (1 - \mathrm{i})z_2 = 4$

$\Rightarrow z_1 + (1 - \mathrm{i})\left(-\frac{1}{2}\mathrm{i}z_1 + \frac{3}{2}\right) = 4$ M1A1

$\Rightarrow z_1 - \frac{1}{2}\mathrm{i}z_1 + \frac{3}{2} + \frac{1}{2}\mathrm{i}^2 z_1 - \frac{3}{2}\mathrm{i} = 4$

$\Rightarrow \frac{1}{2}z_1 - \frac{1}{2}\mathrm{i}z_1 = \frac{5}{2} + \frac{3}{2}\mathrm{i}$

$\Rightarrow z_1 - \mathrm{i}z_1 = 5 + 3\mathrm{i}$ A1

EITHER

Let $z_1 = x + \mathrm{i}y$ (M1)

$\Rightarrow x + \mathrm{i}y - \mathrm{i}x - \mathrm{i}^2 y = 5 + 3\mathrm{i}$

Equate real and imaginary parts M1

$\Rightarrow x + y = 5$

$\underline{-x + y = 3}$

$2y = 8$

$y = 4 \Rightarrow x = 1$ i.e. $z_1 = 1 + 4\mathrm{i}$ A1A1

$z_2 = -\frac{1}{2}\mathrm{i}(1 + 4\mathrm{i}) + \frac{3}{2}$ M1

$z_2 = -\frac{1}{2}\mathrm{i} - 2\mathrm{i}^2 + \frac{3}{2}$

$z_2 = \frac{7}{2} - \frac{1}{2}\mathrm{i}$ A1

OR

$z_1 = \frac{5 + 3\mathrm{i}}{1 - \mathrm{i}}$ M1

$z_1 = \frac{(5 + 3\mathrm{i})(1 + \mathrm{i})}{(1 - \mathrm{i})(1 + \mathrm{i})} \left(= \frac{5 + 8\mathrm{i} - 3}{2}\right)$ M1A1

$z_1 = 1 + 4\mathrm{i}$ A1

$z_2 = -\frac{1}{2}\mathrm{i}(1 + 4\mathrm{i}) + \frac{3}{2}$ M1

$z_2 = -\frac{1}{2}\mathrm{i} - 2\mathrm{i}^2 + \frac{3}{2}$

$z_2 = \frac{7}{2} - \frac{1}{2}\mathrm{i}$ A1

[9]

3. $a^2 + 2\mathrm{i}ab - b^2 = 3 + 4\mathrm{i}$

Equate real and imaginary parts (M1)

$a^2 - b^2 = 3,\ 2ab = 4$ A1

Since $b = \frac{2}{a}$

$\Rightarrow a^2 - \frac{4}{a^2} = 3$ (M1)

$\Rightarrow a^4 - 3a^2 - 4 = 0$ A1

Using factorisation or the quadratic formula (M1)

$\Rightarrow a = \pm 2$

$\Rightarrow b = \pm 1$

$\Rightarrow \sqrt{3 + 4\mathrm{i}} = 2 + \mathrm{i}, -2 - \mathrm{i}$ A1A1

[7]

4. (a) $\mathrm{i}^4 - 5\mathrm{i}^3 + 7\mathrm{i}^2 - 5\mathrm{i} + 6 = 1 + 5\mathrm{i} - 7 - 5\mathrm{i} + 6$ M1A1

$= 0$ AG N0

(b) i root $\Rightarrow$ –i is second root (M1)A1

moreover, $x^4 - 5x^3 + 7x^2 - 5x + 6 = (x - \mathrm{i})(x + \mathrm{i})\,q(x)$

where $q(x) = x^2 - 5x + 6$

finding roots of $q(x)$

the other two roots are 2 and 3 A1A1

Note: Final A1A1 is independent of previous work.

[6]

5. (a) $|z| = z$, $\arg(z) = 0$ A1A1

so $L(z) = \ln z$ AG N0

(b) (i) $L(-1) = \ln 1 + \mathrm{i}\pi = \mathrm{i}\pi$ A1A1 N2

(ii) $L(1 - \mathrm{i}) = \ln\sqrt{2} + \mathrm{i}\frac{7\pi}{4}$ A1A1 N2

(iii) $L(-1 + \mathrm{i}) = \ln\sqrt{2} + \mathrm{i}\frac{3\pi}{4}$ A1 N1

(c) for comparing the product of two of the above results with the third M1

for stating the result $-1 + \mathrm{i} = -1 \times (1 - \mathrm{i})$ and $L(-1 + \mathrm{i}) \neq L(-1) + L(1 - \mathrm{i})$ R1

hence, the property $L(z_1 z_2) = L(z_1) + L(z_2)$

does not hold for all values of z_1 and z_2 AG N0

[9]