1. Given that $z_1 = 2$ and $z_2 = 1 + \text{i}\sqrt{3}$ are roots of the cubic equation $z^3 + bz^2 + cz + d = 0$ where $b, c, d \in \mathbb{R}$,

(a) write down the third root, z_3, of the equation;

(1)

(b) find the values of b, c and d;

(4)

(c) write z_2 and z_3 in the form $r\text{e}^{\text{i}\theta}$.

(3)
(Total 8 marks)

2. Solve the simultaneous equations

$$\text{i}z_1 + 2z_2 = 3$$
$$z_1 + (1 - \text{i})z_2 = 4$$

giving z_1 and z_2 in the form $x + \text{i}y$, where x and y are real.

(Total 9 marks)

3. Given that $(a + b\text{i})^2 = 3 + 4\text{i}$ obtain a pair of simultaneous equations involving a and b. Hence find the two square roots of $3 + 4\text{i}$.

(Total 7 marks)

4. (a) Show that the complex number i is a root of the equation

$$x^4 - 5x^3 + 7x^2 - 5x + 6 = 0.$$

(2)

(b) Find the other roots of this equation.

(4)
(Total 6 marks)

5. If z is a non-zero complex number, we define $L(z)$ by the equation

$$L(z) = \ln|z| + \mathrm{i} \arg(z),\ 0 \leq \arg(z) < 2\pi.$$

(a) Show that when z is a positive real number, $L(z) = \ln z$.

(2)

(b) Use the equation to calculate

(i) $L(-1)$;

(ii) $L(1 - \mathrm{i})$;

(iii) $L(-1 + \quad \mathrm{i})$.

(5)

(c) Hence show that the property $L(z_1 z_2) = L(z_1) + L(z_2)$ does not hold for all values of z_1 and z_2.

(2)

(Total 9 marks)