FORMA	TIVE	OHIZ
TUNMA		UUIZ

name:	 score:	

1. (a) The sum of the first six terms of an arithmetic series is 81. The sum of its first eleven terms is 231. Find the first term and the common difference.

(6)

(b) The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find the first term and the common ratio.

(5)

(c) The r^{th} term of a new series is defined as the product of the r^{th} term of the arithmetic series and the r^{th} term of the geometric series above. Show that the r^{th} term of this new series is $(r+1)2^{r-1}$.

(3)

(d) Using mathematical induction, prove that

$$\sum_{r=1}^{n} (r+1)2^{r-1} = n2^{n}, n \in \mathbb{Z}^{+}.$$

(7)

(Total 21 marks)

2. (a) Find the sum of the infinite geometric sequence $27, -9, 3, -1, \dots$

(3)

(b) Use mathematical induction to prove that for $n \in \mathbb{Z}^+$,

$$a + ar + ar^2 + ... + ar^{n-1} = \frac{a(1-r^n)}{1-r}.$$

(Total 10 marks)