Math HL 1 Quadratic Equations and Functions name: _____

1a. Prove that the equation $3x^2 + 2kx + k - 1 = 0$ has two distinct real roots for all values of $k \in \mathbb{R}$. *[4 marks]*

1b. Find the value of k for which the two roots of the equation are closest together. *[3 marks]*

2. One root of the equation $x^2 + ax + b = 0$ is $2 + 3\mathrm{i}$ where $a, b \in \mathbb{R}$. Find the value of a and the value of b. *[4 marks]*

3. The roots of a quadratic equation $2x^2 + 4x - 1 = 0$ are α and β. *[6 marks]*
Without solving the equation,
(a) find the value of $\alpha^2 + \beta^2$;
(b) find a quadratic equation with roots α^2 and β^2.

4. Show that the quadratic equation $x^2 - (5 - k)x - (k + 2) = 0$ has two distinct real roots for all real values of k. *[4 marks]*

5. One form of *Schwarz's Inequality* states that for any 4 real numbers p, q, r, and s,

$$pr + qs \leq \sqrt{p^2 + q^2} \cdot \sqrt{r^2 + s^2}.$$

Prove Schwarz's Inequality using the following steps.

a. Let $f(x) = (px + r)^2 + (qx + s)^2$. Explain why, for any real number x, $f(x) \geq 0$.

b. Expand $f(x)$, and express the discriminant of $f(x) = 0$ in terms of p, q, r, and s. (Leave your answer in factored form.)

c. What does the fact that $f(x) \geq 0$ tell you about where the graph of $y = f(x)$ is situated in relation to the x-axis? What does this tell you about the roots of the equation $f(x) = 0$? What does it therefore tell you about the discriminant of the equation $f(x) = 0$?

d. Use your answer to the last question of part (c) to prove Schwarz's Inequality.