1. A function f is defined by $f(x) = \dfrac{2x-3}{x-1}$, $x \neq 1$.

(a) Find an expression for $f^{-1}(x)$.

(3)

(b) Solve the equation $\left| f^{-1}(x) \right| = 1 + f^{-1}(x)$.

(3)
(Total 6 marks)

solu. (a) **Note:** Interchange of variables may take place at any stage.
for the inverse, solve for x in

$y = \dfrac{2x-3}{x-1}$

$y(x-1) = 2x - 3$ M1

$yx - 2x = y - 3$

$x(y-2) = y - 3$ (A1)

$x = \dfrac{y-3}{y-2}$

$\Rightarrow f^{-1}(x) = \dfrac{x-3}{x-2}$ $(x \neq 2)$ A1

Note: Do not award final A1 unless written in the form $f^{-1}(x) = ...$

(b) $\pm f^{-1}(x) = 1 + f^{-1}(x)$ leads to

$2\dfrac{x-3}{x-2} = -1$ (M1)A1

$x = \dfrac{8}{3}$ A1

It was pleasing to see a lot of good work with part (a), though some candidates lost marks due to problems with the algebra which led to one or more incorrect values. Regarding part (b), most candidates did not succeed in finding the new intercepts and asymptotes and were unable to apply the absolute value function. A significant number of candidates misread part (b) and took it as the modulus of the graph in part (a).

2. Let $f(x) = \dfrac{4-x^2}{4-\sqrt{x}}$.

(a) State the largest possible domain for f.

(2)

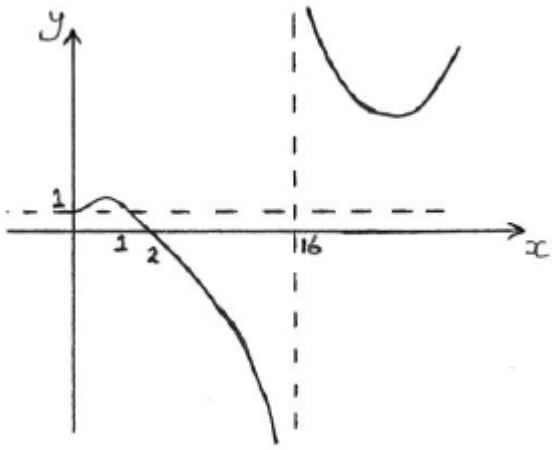
(b) Solve the inequality $f(x) \geq 1$.

(4)
(Total 6 marks)

Solu

2. (a) $x \geq 0$ and $x \neq 16$ A1A1

(b)



graph not to scale (M1)

finding crossing points $\sqrt{x}$

e.g. $4 - x^2 = 4 -$

$x = 0$ or $x = 1$ (A1)

$0 \leq x \leq 1$ or $x > 16$ A1A1

Note: Award M1A1A1A0 for solving the inequality only for the case $x < 16$

Very few completely correct answers were given to this question. Many students found *a* to be 0 and many failed to provide adequate sketches. There were very few correct answers to part (c) although many students were able to obtain partial marks.

3. Find the set of values of x for which $|x - 1| > |2x - 1|$.
(Total 4 marks)

solu. **EITHER**

$|x - 1| > |2x - 1| \Rightarrow (x - 1)^2 > (2x - 1)^2$ M1

$x^2 - 2x + 1 > 4x^2 - 4x + 1$

$3x^2 - 2x < 0$ A1

$0 < x < \frac{2}{3}$ A1A1

N2

Note: Award A1A0 for incorrect inequality signs.

OR

$|x - 1| > |2x - 1|$

$x - 1 = 2x - 1$ $\quad x - 1 = 1 - 2x$ M1A1

$-x = 0$ $\quad 3x = 2$

$x = 0$ $\quad x = \frac{2}{3}$

Note: Award M1 for any attempt to find a critical value. If graphical methods are used, award M1 for correct graphs, A1 for correct values of x.

$0 < x < \frac{2}{3}$ A1A1

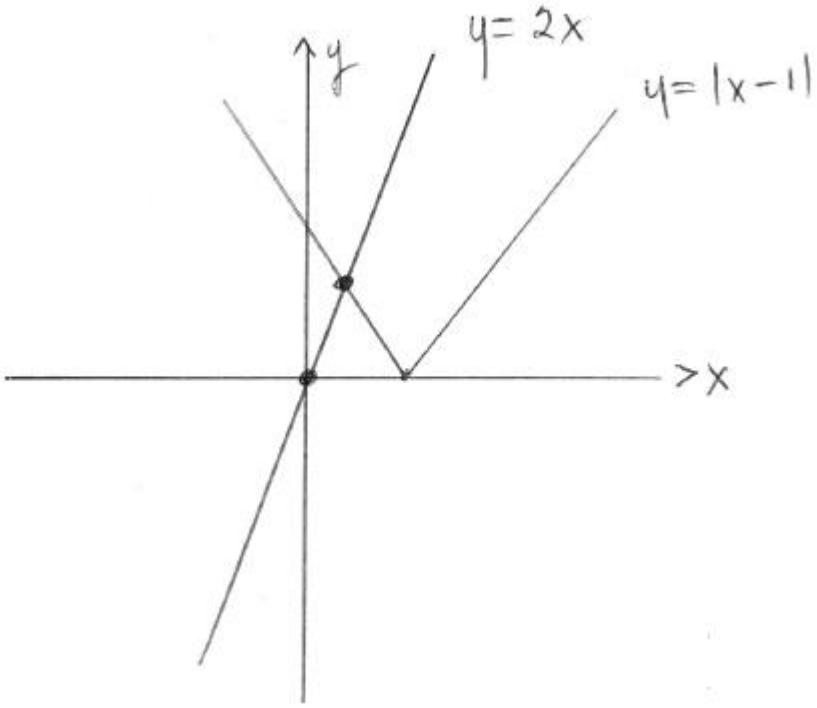
N2

Note: Award A1A0 for incorrect inequality signs

This question was generally well done, with very few candidates calculating $f \circ g$ rather than $g \circ f$.

4. Find all values of x that satisfy the inequality $\frac{2x}{|x-1|} < 1$.

(Total 5 marks)



A1A1

Note: Award A1 for each graph.

$2x = 1 - x \Rightarrow x = \frac{1}{3}$ M1A1

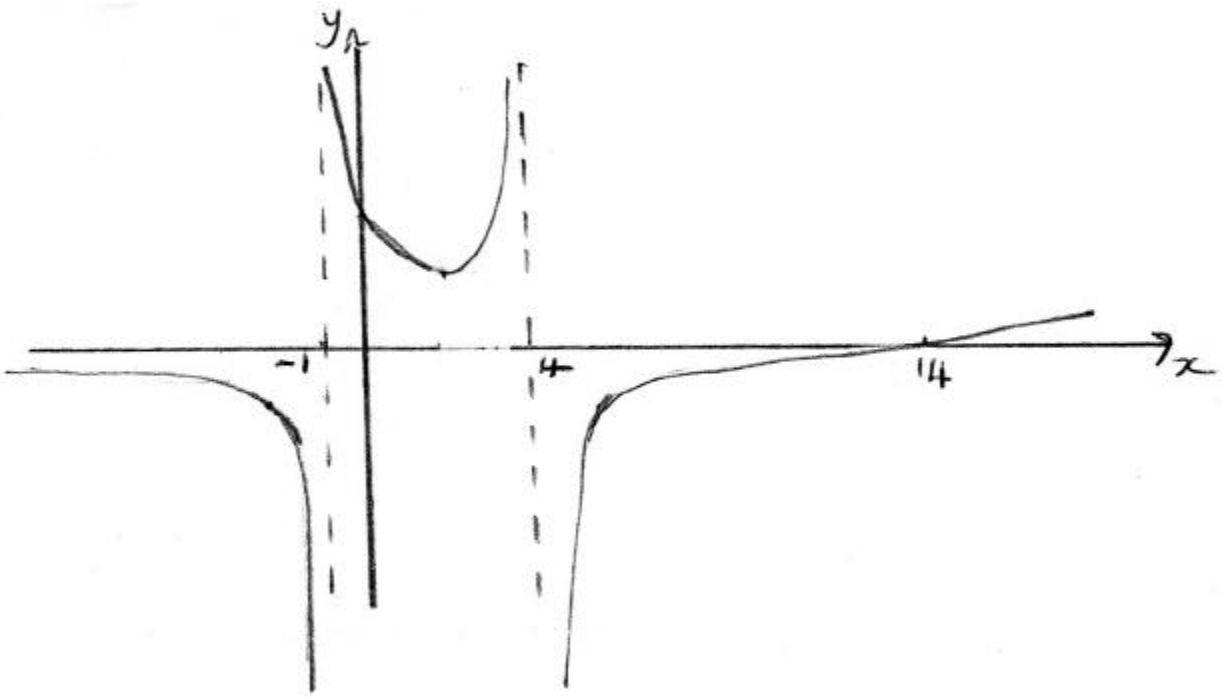
$\therefore x < \frac{1}{3}$ A1

5. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$. Find the set of values of x such that $f(x) \leq g(x)$.

(Total 6 marks)

METHOD 1

Graph of $f(x) - g(x)$ M1



A1A1A1

Note: Award A1 for each branch.

$x < -1$ or $4 < x \leq 14$ A1A1
N3

Note: Each value and inequality sign must be correct.

METHOD 2

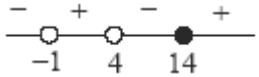
$\frac{x+4}{x+1} - \frac{x-2}{x-4} \leq 0$ M1

$\frac{x^2 - 16 - x^2 + x + 2}{(x+1)(x-4)} \leq 0$

$\frac{x-14}{(x+1)(x-4)} \leq 0$ A1

Critical value of $x = 14$ A1

Other critical values $x = -1$ and $x = 4$ A1



$x < -1$ or $4 < x \leq 14$ A1A1
N3

Note: Each value and inequality sign must be correct.

Many candidates were able to find the reciprocal but many struggled with the second part. Sketches were quite poor in detail.