1. A function f is defined by $f(x) = \dfrac{2x-3}{x-1}$, $x \neq 1$.

(a) Find an expression for $f^{-1}(x)$.

(3)

(b) Solve the equation $\left|f^{-1}(x)\right| = 1 + f^{-1}(x)$.

(3)
(Total 6 marks)

solu. (a) **Note:** Interchange of variables may take place at any stage.
for the inverse, solve for x in

$y = \dfrac{2x-3}{x-1}$

$y(x-1) = 2x-3$ M1

$yx - 2x = y - 3$

$x(y-2) = y-3$ (A1)

$x = \dfrac{y-3}{y-2}$

$\Rightarrow f^{-1}(x) = \dfrac{x-3}{x-2}$ $(x \neq 2)$ A1

Note: Do not award final A1 unless written in the form $f^{-1}(x) = ...$

(b) $\pm f^{-1}(x) = 1 + f^{-1}(x)$ leads to

$2\dfrac{x-3}{x-2} = -1$ (M1)A1

$x = \dfrac{8}{3}$ A1

2. Let $f(x) = \dfrac{4-x^2}{4-\sqrt{x}}$.

(a) State the largest possible domain for f.

(2)

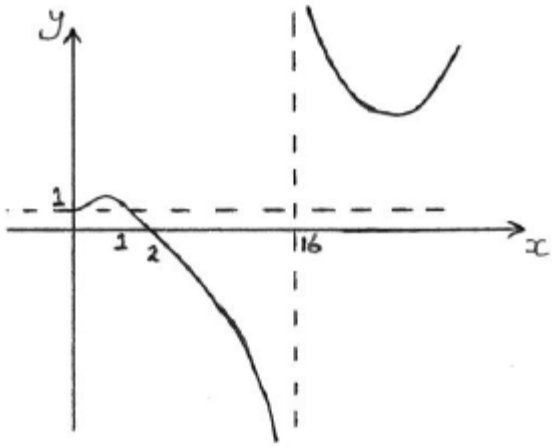
(b) Solve the inequality $f(x) \geq 1$.

(4)
(Total 6 marks)

Solu

2. (a) $x \geq 0$ and $x \neq 16$ A1A1

(b)



graph not to scale (M1)

finding crossing points

e.g. $4 - x^2 = 4 - \sqrt{x}$

$x = 0$ or $x = 1$ (A1)

$0 \leq x \leq 1$ or $x > 16$ A1A1

Note: Award M1A1A1A0 for solving the inequality only for the case $x < 16$

3. Find the set of values of x for which $|x - 1| > |2x - 1|$.
(Total 4 marks)

solu. **EITHER**

$|x - 1| > |2x - 1| \Rightarrow (x - 1)^2 > (2x - 1)^2$ M1

$x^2 - 2x + 1 > 4x^2 - 4x + 1$

$3x^2 - 2x < 0$ A1

$0 < x < \frac{2}{3}$ A1A1

N2

Note: Award A1A0 for incorrect inequality signs.

OR

$|x - 1| > |2x - 1|$

$x - 1 = 2x - 1$ $\quad x - 1 = 1 - 2x$ M1A1

$-x = 0$ $\quad 3x = 2$

$x = 0$ $\quad x = \frac{2}{3}$

Note: Award M1 for any attempt to find a critical value. If graphical methods are used, award M1 for correct graphs, A1 for correct values of x.

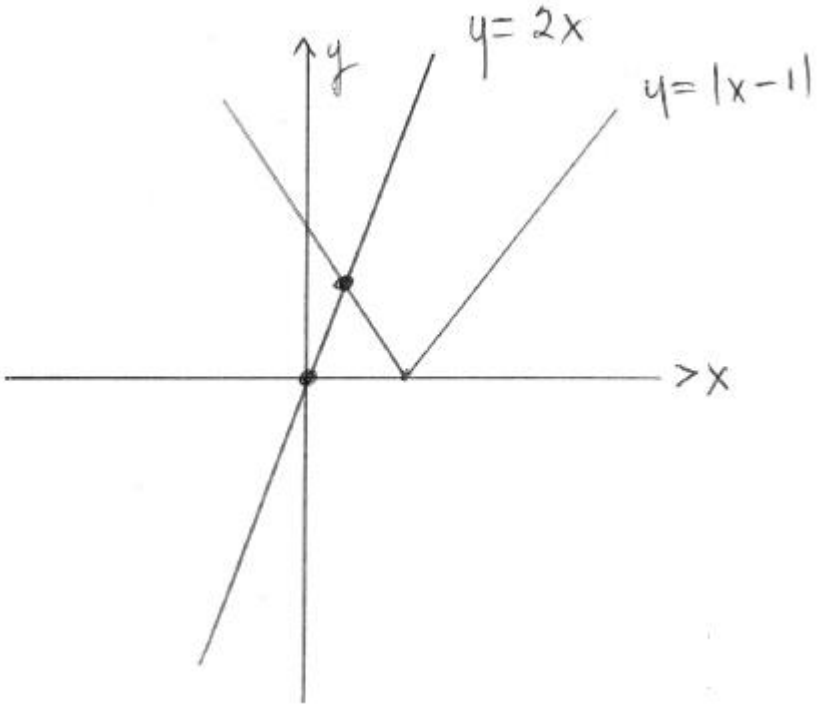
$0 < x < \frac{2}{3}$ A1A1

N2

Note: Award A1A0 for incorrect inequality signs

4. Find all values of x that satisfy the inequality $\frac{2x}{|x-1|} < 1$.

(Total 5 marks)



A1A1

Note: Award A1 for each graph.

$2x = 1 - x \Rightarrow x = \frac{1}{3}$ M1A1

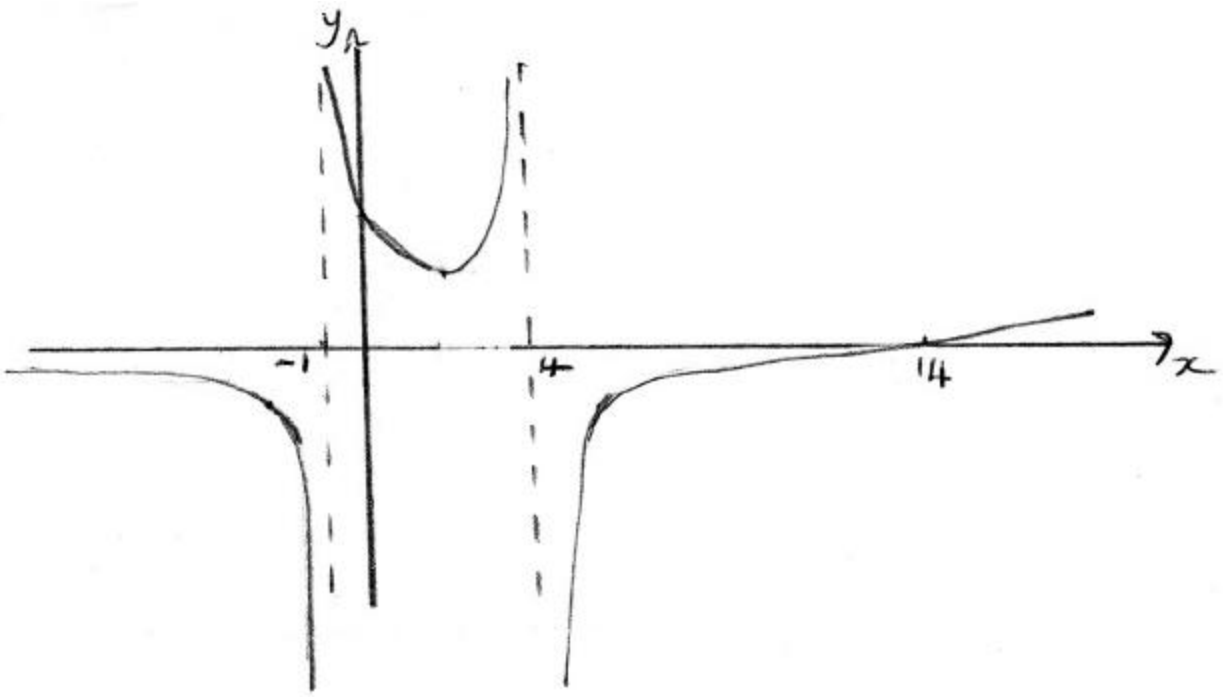
$\therefore x < \frac{1}{3}$ A1

5. Let $f(x) = \frac{x+4}{x+1}$, $x \neq -1$ and $g(x) = \frac{x-2}{x-4}$, $x \neq 4$. Find the set of values of x such that $f(x) \leq g(x)$.

(Total 6 marks)

METHOD 1

Graph of $f(x) - g(x)$ M1



A1A1A1

Note: Award A1 for each branch.

$x < -1$ or $4 < x \leq 14$ A1A1
N3

Note: Each value and inequality sign must be correct.

METHOD 2

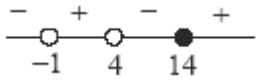
$\frac{x+4}{x+1} - \frac{x-2}{x-4} \leq 0$ M1

$\frac{x^2 - 16 - x^2 + x + 2}{(x+1)(x-4)} \leq 0$

$\frac{x-14}{(x+1)(x-4)} \leq 0$ A1

Critical value of $x = 14$ A1

Other critical values $x = -1$ and $x = 4$ A1



$x < -1$ or $4 < x \leq 14$ A1A1
N3

Note: Each value and inequality sign must be correct.