Oct.31 Math HL

*① $g(x) = \log_4(\log_3(\log_2(x)))$. To the nearest ten-thousandth, $g^{-1}\left(\frac{1}{4}\right) = ?$

$$y = \log_4(\log_3(\log_2 x))$$

$$x = \log_4(\log_3(\log_2 y))$$

$$\frac{1}{4} = \log_4(\log_3(\log_2 y))$$

$$4^{1/4} = \log_3(\log_2 y)$$

$$3^{4^{1/4}} = \log_2 y$$

$$2^{3^{4^{1/4}}} = y$$

$$2\wedge(3\wedge(4\wedge(1/4))) = \boxed{26.5162}$$

② Describe in words the transformations on the parent function $f(x)=\sqrt{x}$ to obtain the function $g(x)=\frac{1}{2}\sqrt{3-x}+2$.

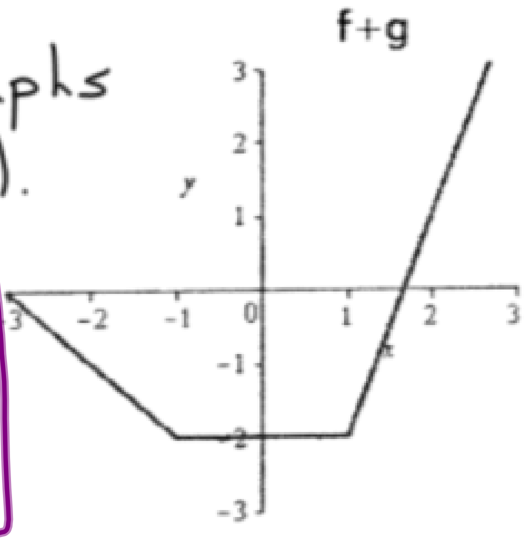
A reflection about the y-axis, followed by a horizontal shift 3 units, then a vertical compression by 1/2, and a vertical shift up 2 units.

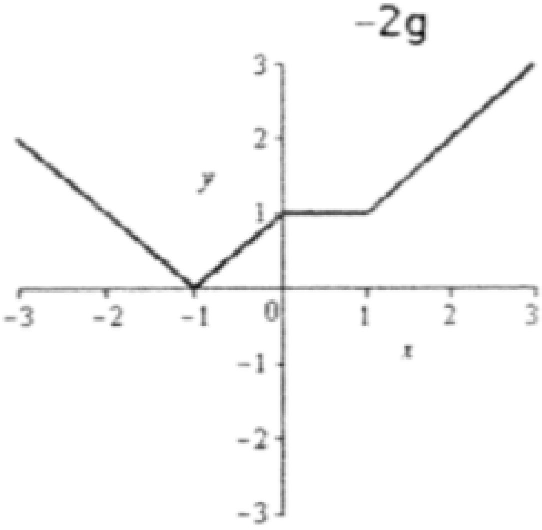
③ Using interval notation, state the domain of $f(x) = \dfrac{\sqrt{3-x}}{\ln(2x)}$.

$$\boxed{(0, \tfrac{1}{2}) \cup (\tfrac{1}{2}, 3]}$$

4 Use the graphs to find $f(0)$.

$\boxed{f(0) = -1.5}$





5 Using the same two graphs above, determine the value of $f(g(2))$.

$\boxed{f(g(2)) = -2}$

⑥ The function $f(x) = K(3-2x+x^4)$ has an inverse function for $x \geq 1$, and $f^{-1}(-3)=1$. Find K.

$$f^{-1}(-3) = 1 \Rightarrow f(1) = -3$$

then, $f(1) = 2K = -3$

$$\therefore \boxed{K = \frac{-3}{2}}$$

7. Let $f(x)=\frac{ax+b}{cx+d}$ where a, b, c, and d are non-zero constants. Express $f^{-1}(x)$ as a single fraction.

$$y=\frac{ax+b}{cx+d} \qquad x=\frac{ay+b}{cy+d}$$

$$cyx+dx=ay+b$$

$$cyx-ay=b-dx$$

$$y(cx-a)=-dx+b$$

$$y=\frac{-dx+b}{cx-a}=\boxed{-\frac{dx-b}{cx-a}}$$

(8) Evaluate $(\log_2 3)(\log_3 4)(\log_4 5)\cdots(\log_{63} 64)$.

change of base formula...

$$\frac{\log \cancel{3}}{\log 2} \cdot \frac{\log \cancel{4}}{\log \cancel{3}} \cdot \frac{\log \cancel{5}}{\log \cancel{4}} \cdots \frac{\log \cancel{63}}{\log \cancel{62}} \cdot \frac{\log 64}{\log \cancel{63}}$$

$$= \log_2 64 = \boxed{6}$$

9) If $\log_a x = 8$, $\log_a y = 6$, and $\log_a z = 4$, find $\log_a \dfrac{\sqrt[4]{y^2 z^5}}{\sqrt[4]{x^3 z^{-2}}}$.

$a^8 = x \qquad a^6 = y \qquad a^4 = z$

$$\log_a \sqrt[4]{\frac{a^{12} \cdot a^{20} \cdot a^8}{a^{24}}}$$

$$= \log_a \sqrt[4]{\frac{a^{40}}{a^{24}}} = \log_a \sqrt[4]{a^{16}}$$

$$= \log_a a^4 = \boxed{4}$$

10 Solve for x: $x^5e^x + 3x^4e^x = 4x^3e^x$.

$$x^5e^x + 3x^4e^x - 4x^3e^x = 0$$

$$e^x(x^5 + 3x^4 - 4x^3) = 0$$

$$e^x \cdot x^3(x^2 + 3x - 4) = 0$$

$$e^x \cdot x^3(x+4)(x-1) = 0$$

by the zero product property, $\boxed{x = 1, -4, \text{ or } 0}$

(11) If $\log(x^2y^3) = a$ and $\log\left(\frac{x}{y}\right) = b$, then in terms of only a and b, $\log x = ?$

first solve both eq's for $\log y$:

$\log x^2 + \log y^3 = a$

$2\log x + 3\log y = a$

$3\log y = a - 2\log x$

$\log y = \frac{a - 2\log x}{3}$

$\log x - \log y = b$

$-\log y = b - \log x$

$\log y = \log x - b$

then set these equal to each other: $\frac{a-2\log x}{3} = \log x - b$

$a - 2\log x = 3\log x - 3b$

$a + 3b = 5\log x$

$\therefore \log x = \boxed{\frac{a+3b}{5}}$

(12) Find the remainder when $3x^7+5x^6+2x^4-7x^3+x^2+4x+9$ is divided by $2x+4$.

$2x+4=0$

$2x=-4$

$x=-2 \longrightarrow$

$$
\begin{array}{r|rrrrrrrr}
-2 & 3 & 5 & 0 & 2 & -7 & 1 & 4 & 9 \\
 & & -6 & 2 & -4 & 4 & 6 & -14 & 20 \\
\hline
 & 3 & -1 & 2 & -2 & -3 & 7 & -10 & \boxed{29}
\end{array}
$$

13 Suppose $f(x)$ is a fifth degree polynomial with rational coefficients, no constant term, at least one irrational root, and at least one imaginary root. List all of the rational roots of $f(x)$.

0

14 Solve for x: $2 = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$

$x^{x^{x^{x^{\cdots}}}} = ?$

Let it be that $y = x^{x^{x^{x^{\cdots}}}}$,

then $y = x^y$ and $\ln y = \ln x^y$

...but $y = 2$

so, $\ln 2 = 2 \ln x$

$\frac{\ln 2}{2} = \ln x$

$e^{\frac{\ln 2}{2}} = x = e^{\ln 2^{\frac{1}{2}}} = 2^{\frac{1}{2}} = \boxed{\sqrt{2}}$