NOTES...

Use induction to prove:

$$\sum_{i=1}^{n} (2i-1) = n^2 , \quad n \in \mathbb{Z}^+ .$$

step1: Show that it is true for $n=1$. (base case)

$$\sum_{i=1}^{1} (2i-1) = 1 \quad \Rightarrow \quad 1^2 = 1 \ ✓$$

step2: Assume it is true for some $n=k$.

$$\sum_{i=1}^{k} (2i-1) = k^2$$

step3: (inductive step) Show that assumption from step 2 implies that proof statement is true for $n=k+1$.

$$\sum_{i=1}^{k+1} (2i-1) = \sum_{i=1}^{k} (2i-1) + 2(k+1) - 1$$

$$= k^2 + 2k + 2 - 1$$

$$= k^2 + 2k + 1$$

$$= (k+1)^2$$

$$\therefore \sum_{i=1}^{n} (2i-1) = n^2 , \quad n \in \mathbb{Z}^+ .$$

FORMATIVE:

F1. Prove by mathematical induction that, for $n \in \mathbb{Z}^+$,

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}.$$

(Total 8 marks)

F2. Use mathematical induction to prove that $5^n + 9^n + 2$ is divisible by 4, for $n \in \mathbb{Z}^+$.

(Total 9 marks)

PRACTICE:

P1. (a) Consider the following sequence of equations.

$$1 \times 2 = \frac{1}{3}(1 \times 2 \times 3),$$

$$1 \times 2 + 2 \times 3 = \frac{1}{3}(2 \times 3 \times 4),$$

$$1 \times 2 + 2 \times 3 + 3 \times 4 = \frac{1}{3}(3 \times 4 \times 5),$$

.... .

(i) Formulate a conjecture for the n^{th} equation in the sequence.

(ii) Verify your conjecture for $n = 4$.

(2)

(b) A sequence of numbers has the n^{th} term given by $u_n = 2^n + 3$, $n \in \mathbb{Z}^+$. Bill conjectures that all members of the sequence are prime numbers. Show that Bill's conjecture is false.

(2)

(c) Use mathematical induction to prove that $5 \times 7^n + 1$ is divisible by 6 for all $n \in \mathbb{Z}^+$.

(6)

(Total 10 marks)

P2. Prove by mathematical induction $\sum_{r=1}^{n} r(r!) = (n+1)! - 1,\ n \in \mathbb{Z}^{+}$.

(Total 8 marks)