INDUCTION PRACTICE AUGUST 15

Use the principle of mathematical induction to prove each.

1 A sequence is defined by $u_n = 2\times 3^{n-1}$.

Use the principle of mathematical induction to prove that $u_1 + u_2 + \ldots + u_n = 3^n - 1$. *[6 marks]*

2 Prove by induction that:

$$\frac{1}{1\times 2}+\frac{1}{2\times 3}+\frac{1}{3\times 4}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}.$$ *[6 marks]*

3 Prove by induction that:

$$\frac{1}{1\times 3}+\frac{1}{3\times 5}+\frac{1}{5\times 7}+\ldots+\frac{1}{(2n-1)\times(2n+1)}=\frac{n}{2n+1}.$$ *[6 marks]*

4 Prove that $1\times 1!+2\times 2!+3\times 3!\ldots+n\times n!=(n+1)!-1$. *[6 marks]*

5 Use the principle of mathematical induction to show:

$$1^2-2^2+3^2-4^2+\ldots+(-1)^{n-1}n^2=(-1)^{n-1}\frac{n(n+1)}{2}$$ *[6 marks]*

6 Prove that $(n+1)+(n+2)+(n+3)+\ldots+(2n)=\frac{1}{2}n(3n+1)$. *[6 marks]*

7 Prove that $\sum_{k=1}^{n} k2^k = (n-1)2^{n+1}+2$. *[6 marks]*

8 Show that $5^n - 1$ is divisible by 4 for all $n \in \mathbb{N}$. *[8 marks]*

9 Show that $4^n - 1$ is divisible by 3 for all $n \geq 1$. *[8 marks]*

10 Show that $7^n - 3^n$ is divisible by 4 for all $n \in \mathbb{N}$ integers. *[8 marks]*

11 Use induction to prove that $30^n - 6^n$ is divisible by 12 for all integers $n \geq 0$. *[8 marks]*

12 Show using induction that $n^3 - n$ is divisible by 6 for all integers $n \geq 1$. *[9 marks]*

13 Using the principle of mathematical induction, prove that $n(n^2+5)$ is divisible by 6 for all integers $n \geq 1$. *[9 marks]*

14 Use induction to show that $7^n - 4^n - 3^n$ is divisible by 12 for all $n \in \mathbb{Z}^+$. *[9 marks]*

15 Prove, using the principle of mathematical induction, that $3^{2n+2} - 8n - 9$ is divisible by 64 for all positive integers n. *[8 marks]*

16 Show that the sum of the cubes of any three consecutive integers is divisible by 9. *[8 marks]*

17 Find the set of positive integers for which $3^n > n^3$ and prove your claim by induction. *[8 marks]*

18 $2^n > 1 + n$ for all $n > 1$ *[8 marks]*

19 $2^n > n^2$ for all $n \geq 4$ *[8 marks]*

20 $n! > 2^n$ for all $n \geq 4$ *[8 marks]*

21 $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > \sqrt{n}$ for all $n > 1$. *[8 marks]*

22 $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \ldots + \frac{1}{\sqrt{n}} > 2(\sqrt{n+1} - 1)$ for all $n \geq 1$. *[8 marks]*

23 Find the smallest integer N for which $3^N < N!$ Prove that $3^n < n!$ for all $n \geq$ N. *[10 marks]*

24 Show that $(1+x)^n \geq 1 + nx$ for $n \in \mathbb{N}$ and $x \in \mathbb{R}$. *[8 marks]*