

1. The sum, S_n , of the first n terms of a geometric sequence, whose n^{th} term is u_n , is given by

$$S_n = \frac{7^n - a^n}{7^n}, \text{ where } a > 0.$$

- (a) Find an expression for u_n . (2)
- (b) Find the first term and common ratio of the sequence. (4)
- (c) Consider the sum to infinity of the sequence.
- (i) Determine the values of a such that the sum to infinity exists.
- (ii) Find the sum to infinity when it exists. (2)

(Total 8 marks)

2. The common ratio of the terms in a geometric series is 2^x .

- (a) State the set of values of x for which the sum to infinity of the series exists. (2)
- (b) If the first term of the series is 35, find the value of x for which the sum to infinity is 40. (4)

(Total 6 marks)

3. A circular disc is cut into twelve sectors whose areas are in an arithmetic sequence. The angle of the largest sector is twice the angle of the smallest sector.

Find the size of the angle of the smallest sector.

(Total 5 marks)

4. The interior of a circle of radius 2 cm is divided into an infinite number of sectors. The areas of these sectors form a geometric sequence with common ratio k . The angle of the first sector is θ radians.

(a) Show that $\theta = 2\pi(1 - k)$.

(5)

- (b) The perimeter of the third sector is half the perimeter of the first sector.

Find the value of k and of θ .

(7)

(Total 12 marks)

5. An arithmetic sequence has first term a and common difference d , $d \neq 0$. The 3rd, 4th and 7th terms of the arithmetic sequence are the first three terms of a geometric sequence.

(a) Show that $a = -\frac{3}{2}d$.

(3)

- (b) Show that the 4th term of the geometric sequence is the 16th term of the arithmetic sequence.

(5)

(Total 8 marks)

6. A geometric sequence u_1, u_2, u_3, \dots has $u_1 = 27$ and a sum to infinity of $\frac{81}{2}$.

- (a) Find the common ratio of the geometric sequence.

(2)

An arithmetic sequence v_1, v_2, v_3, \dots is such that $v_2 = u_2$ and $v_4 = u_4$.

- (b) Find the greatest value of N such that $\sum_{n=1}^N v_n > 0$.

(5)

(Total 7 marks)