1. **METHOD 1**

(a) $u_n = S_n - S_{n-1}$ (M1)

$= \frac{7^n - a^n}{7^n} - \frac{7^{n-1} - a^{n-1}}{7^{n-1}}$ A1

(b) **EITHER**

$u_1 = 1 - \frac{a}{7}$ A1

$u_2 = 1 - \frac{a^2}{7^2} - \left(1 - \frac{a}{7}\right)$ M1

$= \frac{a}{7}\left(1 - \frac{a}{7}\right)$ A1

common ratio = $\frac{a}{7}$ A1

OR

$u_n = 1 - \left(\frac{a}{7}\right)^n - 1 + \left(\frac{a}{7}\right)^{n-1}$ M1

$= \left(\frac{a}{7}\right)^{n-1}\left(1 - \frac{a}{7}\right)$

$= \frac{7-a}{7}\left(\frac{a}{7}\right)^{n-1}$ A1

$u_1 = \frac{7-a}{7}$, common ratio = $\frac{a}{7}$ A1A1

(c) (i) $0 < a < 7$ (accept $a < 7$) A1

(ii) 1 A1

METHOD 2

(a) $u_n = br^{n-1} = \left(\frac{7-a}{7}\right)\left(\frac{a}{7}\right)^{n-1}$ A1A1

(b) for a GP with first term b and common ratio r

$S_n = \frac{b(1-r^n)}{1-r} = \left(\frac{b}{1-r}\right) - \left(\frac{b}{1-r}\right)r^n$ M1

as $S_n = \frac{7^n - a^n}{7^n} = 1 - \left(\frac{a}{7}\right)^n$

comparing both expressions M1

$\frac{b}{1-r} = 1$ and $r = \frac{a}{7}$

$b = 1 - \frac{a}{7} = \frac{7-a}{7}$

$u_1 = b = \frac{7-a}{7}$, common ratio = $r = \frac{a}{7}$ A1A1

Note: Award method marks if the expressions for b and r are deduced in part (a).

(c) (i) $0 < a < 7$ (accept $a < 7$) A1

(ii) 1 A1

[8]

2. (a) $0 < 2^x < 1$ (M1)

$x < 0$ A1 N2

(b) $\frac{35}{1-r} = 40$ M1

$\Rightarrow 40 - 40 \times r = 35$

$\Rightarrow -40 \times r = -5$ (A1)

$\Rightarrow r = 2^x = \frac{1}{8}$ A1

$\Rightarrow x = \log_2 \frac{1}{8}$ $(= -3)$ A1

Note: The substitution $r = 2^x$ may be seen at any stage in the solution.

[6]

3. **METHOD 1**

If the areas are in arithmetic sequence, then so are the angles. (M1)

$\Rightarrow S_n = \frac{n}{2}(a+l) \Rightarrow \frac{12}{2}(\theta + 2\theta) = 18\theta$ M1A1

$\Rightarrow 18\theta = 2\pi$ (A1)

$\theta = \frac{\pi}{9}$ (accept 20°) A1

METHOD 2

$a_{12} = 2a_1$ (M1)

$\frac{12}{2}(a_1 + 2a_1) = \pi r^2$ M1A1

$3a_1 = \frac{\pi r^2}{6}$

$\frac{3}{2} r^2 \theta = \frac{\pi r^2}{6}$ (A1)

$\theta = \frac{2\pi}{18} = \frac{\pi}{9}$ (accept 20°) A1

METHOD 3

Let smallest angle = a, common difference = d

$a + 11d = 2a$ (M1)

$a = 11d$ A1

$S_n = \frac{12}{2}(2a + 11d) = 2\pi$ M1

$6(2a + a) = 2\pi$ (A1)

$18a = 2\pi$

$a = \frac{\pi}{9}$ (accept 20°) A1

[5]

4. (a) the area of the first sector is $\frac{1}{2}2^2\theta$ (A1)

the sequence of areas is $2\theta, 2k\theta, 2k^2\theta...$ (A1)

the sum of these areas is $2\theta(1 + k + k^2 + ...)$ (M1)

$= \frac{2\theta}{1-k} = 4\pi$ M1A1

hence $\theta = 2\pi(1 - k)$ AG

Note: Accept solutions where candidates deal with angles instead of area.

(b) the perimeter of the first sector is $4 + 2\theta$ (A1)

the perimeter of the third sector is $4 + 2k^2\theta$ (A1)

the given condition is $4 + 2k^2\theta = 2 + \theta$ M1

which simplifies to $2 = \theta(1 - 2k^2)$ A1

eliminating θ, obtain cubic in k: $\pi(1 - k)(1 - 2k^2) - 1 = 0$ or equivalent A1

solve for $k = 0.456$ and then $\theta = 3.42$ A1A1

[12]

5. (a) let the first three terms of the geometric sequence be given by $u_1, u_1 r, u_1 r^2$

$\therefore\ u_1 = a + 2d, u_1 r = a + 3d$ and $u_1 r^2 = a + 6d$ (M1)

$\frac{a+6d}{a+3d} = \frac{a+3d}{a+2d}$ A1

$a^2 + 8ad + 12d^2 = a^2 + 6ad + 9d^2$ A1

$2a + 3d = 0$

$a = -\frac{3}{2}d$ AG

(b) $u_1 = \frac{d}{2}, u_1 r = \frac{3d}{2}, \left(u_1 r^2 = \frac{9d}{2}\right)$ M1

$r = 3$ A1

geometric 4th term $u_1 r^3 = \frac{27d}{2}$ A1

arithmetic 16th term $a + 15d = -\frac{3}{2}d + 15d$ M1

$= \frac{27d}{2}$ A1

Note: Accept alternative methods.

[8]

6. (a) $u_1 = 27$

$\frac{81}{2} = \frac{27}{1-r}$ M1

$r = \frac{1}{3}$ A1

(b) $v_2 = 9$

$v_4 = 1$

$2d = -8 \Rightarrow d = -4$ (A1)

$v_1 = 13$ (A1)

$\frac{N}{2}(2 \times 13 - 4(N-1)) > 0$ (accept equality) M1

$\frac{N}{2}(30 - 4N) > 0$

$N(15 - 2N) > 0$

$N < 7.5$ (M1)

$N = 7$ A1

Note: $13 + 9 + 5 + 1 - 3 - 7 - 11 > 0 \Rightarrow N = 7$ or equivalent receives full marks.

[7]