Practice Problem Solutions (Aug. 7, 2018)

P1. (a) (i) $1 \times 2 + 2 \times 3 + ... + n(n+1) = \frac{1}{3}n(n+1)(n+2)$ R1

(ii) LHS = 40; RHS = 40 A1

(b) the sequence of values are:
5, 7, 11, 19, 35 … or an example A1
35 is not prime, so Bill′s conjecture is false R1AG

(c) P(n) : $5 \times 7^n + 1$ is divisible by 6
P(1): 36 is divisible by 6 $\Rightarrow$ P(1) true A1
assume P(k) is true ($5 \times 7^k + 1 = 6r$) M1

Note: Do **not** award M1 for statement starting 'let $n = k$'.
Subsequent marks are independent of this M1.

consider $5 \times 7^{k+1} + 1$ M1
$= 7(6r - 1) + 1$ (A1)
$= 6(7r - 1) \Rightarrow$ P(k + 1) is true A1
P(1) true and P(k) true $\Rightarrow$ P(k + 1) true, so by MI P(n) is true for all $n \in \mathbb{Z}^+$ R1

Note: Only award R1 if there is consideration of P(1), P(k) and P (k + 1) in the final statement.

Only award R1 if at least one of the two preceding A marks has been awarded.

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P2. let $n = 1$
LHS = $1 \times 1! = 1$
RHS = $(1 + 1)! - 1 = 2 - 1 = 1$
hence true for $n = 1$ R1
assume true for $n = k$

$\sum_{r=1}^{k} r(r!) = (k+1)! - 1$ M1

$\sum_{r=1}^{k+1} r(r!) = (k+1)! - 1 + (k+1) \times (k+1)!$ M1A1

$= (k+1)!(1 + k + 1) - 1$
$= (k+1)!(k+2) - 1$ A1
$= (k+2)! - 1$ A1
hence if true for $n = k$, true for $n = k + 1$ R1
since the result is true for $n = 1$ and P (k) $\Rightarrow$ P(k + 1) the result is proved by mathematical induction $\forall n \in \mathbb{Z}^+$ R1

[8]